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# Serviceability Analysis of Algorithms for Diagnosing Technical Objects

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***Abstract*** **– Problems on imitation modelling of algorithms of identification of technical objects are discussed in this article. The authors offer effective method on how to use a priori information of the technical objects, which is collected during their project and exploitation stages. New methods are offered on how to formalize a priori information and how to use it in the computer-controlled algorithms. They are based on the mathematical models, which imitate typical operating conditions of technical objects. These mathematical models are built upon transformation functions of digital models, which are equivalent to transformation functions of the technical objects. There are difficulties on obtaining numerical values for parameters of technical objects. New methods must be used for developing effective software tools to solve problems of identification for technical objects.**

***Keywords*** **– Algorithm, identification, operator, simulation model, transformation function, technical object.**

## I. Problem Analysis and Task Substantiation

Computerized automatic control allows tracking the object position and developing the necessary control signals in emergencies. For example, nuclear power plants, airplanes and other objects are defined as automated control systems. Detailed analyses have been made of automated control and management systems for the technical objects [1]. The current article is devoted to the problem of applying mathematical models to the management systems.

At each moment in time (t1, t2, ...), the current state of the technical object control and management system is based on the arithmetic equation solutions, according to signal measurements. Mathematically, this can be illustrated by the equation:

$$Vx + Ly = y(t) \; t \in (t_1, t_2, ..., t_r)$$

$$Vx = \sum_{i=1}^{n} \alpha_i \cdot x[t-(m+k-i)T]; Ly = \sum_{j=0}^{n-1} \beta_j \cdot y[t-(n-j)\cdot T] \quad . \quad (1)$$

It can be written in matrix form:

$$\bar{\theta} = U^{-1} = \begin{bmatrix} \bar{\alpha} \\ \bar{\xi} \end{bmatrix}; \quad U = [X \; Y]; \quad (2)$$

The object $\bar{\theta}$, is an abstract set of numbers unrelated to its real state. Therefore, it cannot be directly used for diagnosing the object. Special decrypting operations need to be done, and according to the data available to the authors, the solution to this problem has not been previously addressed. The technical facilities of automated control systems and diagnostic computer programs for the improvement of mathematical models remain a challenge.

The system (1) consists of discrete operator $D_M(z)$ that may be expressed as a fraction of two polynomial functions:

$$D_M(z) = \frac{R_M(z)}{B_M(z)} = \frac{y(z)}{x(z)} \quad (3)$$

united by a series of quantum signals (4)

$$x(t) \Rightarrow \Phi z(T) * x(t) \Rightarrow \{x(iT\}$$
$$y(t) \Rightarrow \Phi z(T) * y(t) \Rightarrow \{y(iT\} \quad (4)$$

The equations (3), (4) represent a permanent item with one input and one output for linear objects. The analysis can be extended to objects that are more complex. The control algorithm can be formulated as the vector (2) that is obtained by using object transformation function (TF) $W(p)$.

$$\bar{\theta} \Rightarrow \hat{W}(p) \quad (5)$$

The relation above results from (3) that in the discrete Laplace field may be written as follows:

$$y(z) \cdot B_M(z) - x(z) \cdot R_M(z) = 0 \quad . \quad (6)$$

According to [2] and [3], the object analog TF $W(p)$ may be expressed in terms of equivalent simulation model (IM). An analogue signal of quantization period T is used for linking analog and discrete operators in expression (7). This is denoted as discretization operators $\Phi z(T)$, which can be used as object mathematical formalization of information used for the input control and diagnostic algorithms. The product information is presented by subject TF $W(p)$. The storage and operation parameters of TF can be changed. In the technical documentation they are presented as the nominal parameter values, and the permissible deviations from these values. Storage and operation time parameters $W(p)$ can be changed. The transformation function (TF) $W(p)$ of analog conversion operator in the discrete form

$$\Phi z(T) * W(p) \Rightarrow D_M(z) \quad (7)$$

makes it possible to carry out numerical modeling of operations in accordance with the work [3] and to modify object dynamic nature of information in numerical form.

Another important advantage of using this method is the ability to monitor the performance of a given operator. This can be explained by the discrete form (7) for the Z–modifications that makes it possible to obtain parameters of $W(p)$ and the numerical model of the operator's analytical form $D_M(z)$. This enables one to optimize the computer software for technical object automatic control and diagnostic systems. It may be done automatically if operators start deploying basic line of fractional amounts, each possessing its own corresponding TF partial elemental fraction $W(p)$. Handbooks of TF $W(p)$ complex calculations (7) provide the results in tabular form.

In general, the following operations need to be done:

1) to identify the roots of the analogue characterizing polynomial

$$Qw(p) \in W(p); \quad Qw(p) = \prod_{i=1}^{n} (p + a_i) \tag{8}$$

i.e., complex variable function poles;

2) to convert the TF as the elemental fraction sum at the previous operation of all poles;

$$W(p) = \frac{Rw(p)}{Qw(p)} = \sum_{i=1}^{nW} \left[ \frac{A_i}{(p + a_i)} \right] \quad Qw(p) = \prod_{i}^{nW} (p + a_i) \;; \tag{9}$$

3) to determine the distribution coefficients (as defined TF operator $W(p)$ functions) by the well-known formulas of Korn [5-6-8], obtained without the use of zero values

$$Aw_i = \frac{Rw(p = -a_i)}{\prod\limits_{i=1(j \neq i)}^{n} d_{ij}}; \quad d_{ij} = (a_j - a_i); \tag{10}$$

4) to use Z-transformation for each formula (9) fraction by applying the standard formula:

$$\Phi z(T) * \left[ \frac{A_i}{(p + a_i)} \right] = \frac{zA_i}{z - \beta_i} \;; \tag{11}$$

This non-linear operation is carried out with each term of the sum

$$\Phi z(T) * W(p) \Rightarrow \sum_{i=1}^{n} \Phi z(T) * \left[ \frac{Aw_i}{(p + a_i)} \right] = \sum_{i=1}^{nW} \left[ \frac{Aw_i z}{(p - \beta_i)} \right] \;; \tag{12}$$

5) to find the common denominator for the sum (12), and as a result we obtain:

$$\Phi z(T) * W(p) \Rightarrow \left\{ D_M(z) = \frac{R_M(z)}{B_M(z)} \right\} \tag{13}$$

The result is a polynomial numerator for (3) and (6).

Digital model (7) is obtained directly from the modification of the appropriate analogue [2], [3]:

$$B_M(z) \Rightarrow \Phi z(T) * Qw(p); B_M(z) = \prod_{i=1}^{nW} (z - \beta_i) = \sum_{i=0}^{nW} \xi_i z^{nW-i} \tag{14}$$

$$a_i \in Qw(p); \quad \beta_i \in B_M(z) \quad \beta_i = \exp(-a_i \cdot T).$$

The proposed modification can be used for computer program automatic control and diagnostic systems.

## II. Simulation Models for the Parameter Control Algorithms

Free choice time discretized form does not allow obtaining accurate analytical forms of parameters $W(p)$ and $D_M(z)$ because of the potential non-compliance. Formulas (8) - (14) make it possible to formalize the object information and to use it for automated diagnostic control system computer software algorithms. This information can be found in the technical description of the object (manuals) as a parameter $W(p)$ nominal value. Their fluctuated values are allowed when the object is able to function normally. This fluctuation field can be transformed according to operator $D_M(z)$ of digital models of the simulated object of (7) under normal operating conditions (no need of producing control signals for normalization). The proposed modifications of the operations (10) - (21) are mathematically correct, feasible and measurement error will not depend on the parameter variation levels. The following objects can be expressed by the IF operator $\{D_{Mi}(z)\}$ parameter vectors, which are reflected by the formula (2) vector $\{\bar{\theta}_j\}$ and its deviation field $\{\Delta\bar{\theta}_i\}$. In this case, the control algorithm is based on the solution of equation (2). The resulting difference is compared to the tolerances and is defined by the position of the conclusion in the form of "valid - invalid".

Discrete approximation error can be significant when between the matrix U columns ther is a linear link. Then system (2) does not have a solution and the inverse matrix $U^{-1}$ acts as a background amplifier. To reduce this effect, new unknown parameters must be included in the stochastic model [8], [9].

In order to reduce discrete approximation error, it is desirable to use methods recommended in [2], [3] based on interpolation formulas for series of data (4). They restore almost full analog signal forms. The mathematical formalization is used as fictitious digital filter transformation functions $Int^{(k)}(p,z)$. The simulation model (IM) (7) may be described as follows:

$$D_M(z) = \Phi z(T) * \{Int^{(k)}(p,z) \cdot W(p)\} \tag{15}$$

If the approximation is by rectangle, then

$$Int^{(0)}(p,z) = \frac{(z-1)}{z\ p}; \qquad (16)$$

but with trapezes

$$Int^{(1)}(p,z) = \frac{(z-1)^2}{zTp^2} \qquad (17)$$

Practically any interpolation formula can be found by the operator $Int^{(k)}(p,z)$. In practical tasks, the operator $Int^{(k)}(p,z)$ does not affect the characteristic polynomial $B_M(z) \in D_M(z)$ (14), and IM dynamics defined by (15) for a particular item of poles.

$$\Phi z(T) * Qw(p) \Rightarrow B_M(z) \quad B_M(z) \in \overline{\xi} \qquad (18)$$

$$\Phi\ z(T)\ *\ [p_i = (-a_i)] \Rightarrow \beta_i;$$

$$\{p_i = (-a_i)\} \in Qw(p)\ \beta_i = -\exp(-a_i \cdot T) \qquad (19)$$

## III. Calculation Example of a Simulation Model

Let us consider the calculation example of a simulation model based on computer-assisted automatic flight control. Let us address in more detail the route channel analog signal filter in flight control system of automatic landing mode. Filter transformation function can be written as follows:

$$W(p) = k\frac{T_1 p^2 + 2T_1\xi_1 p + 1}{(T_2^2 p^2 + 2T_2\xi_2 p + 1)(T_3 p + 1)} \qquad (20)$$

The following parameter values are used:

$$T_1 = 0.2\,\text{sec};\ \ T_2 = 0.4\,\text{sec};\ \ T_3 = 0.09\,\text{sec};$$
$$\xi_1 = 1;\ \ \xi_2 = 0.5$$

Let us consider two variants of IM (9) and (10) with T=0.05 sec.

$$D^{(0)}{}_M(z) = \Phi z(T) * \{Int^{(0)}(p,z) \cdot W(p)\} \qquad (21)$$

$$D^{(1)}{}_M(z) = \Phi z(T) * \{Int^{(1)}(p,z) \cdot W(p)\} \qquad (22)$$

$$\overline{\xi} = \begin{bmatrix} 1 \\ -1.25 \\ 2.165 \\ -11.11 \end{bmatrix} \qquad (23)$$

$$\overline{\alpha}_0 = \begin{bmatrix} 2.778 \\ -0.327 \\ -0.184 \\ -0.033 \\ 0.36 \end{bmatrix} \qquad (24)$$

$$\overline{\alpha}_1 = \begin{bmatrix} 2.778 \\ 0.0294 \\ -0.2534 \\ 0.00297 \\ -0.0324 \\ 0.361 \end{bmatrix} \qquad (25)$$

IM characterizing polynomial vector $\overline{\xi}$ coefficients can be determined by (19). Formulas (13), (19) enable the identification of the coefficients $D_M(z)$ (3). Variants (21) and (22) are summarized in (28) and (29). The values of the coefficients given are obtained by (15). The values are shown in Tables I, II. Coefficient values depending on the quantization period are shown in Fig. 1.

$$\left.\begin{array}{l} d_{0,0} = -v_2 - e^{-g_3T}(v_2 - v_1) + e^{2g_1T}(a_3 + a_4) + 2a_3 e^{g_1T}\cos g_2 T + \\ \quad + 2a_4 e^{-g_3T} e^{g_1T}\cos g_2 T \\ d_{0,1} = v_2 e^{-g_3T} - a_3 e^{2g_1T} - e^{2g_1T} e^{-g_3T} \\ d_{0,2} = v_2 - v_1\left(1 + e^{-g_3T}\right) - a_3 - 2e^{g_1T}\cos g_2 T(a_3 - a_4) - a_4 e^{-g_3T} \end{array}\right\}$$

$$d_{1,0} = \frac{1}{T}\left[-v_2 e^{-g_3T} + a_3 e^{2g_1T} + a_4 e^{2g_1T} e^{-b_3} - a_5 e^{2g_1T} e^{-g_3T}\right];$$

$$d_{1,1} = \frac{1}{T}\begin{bmatrix} 2v_2 e^{-g_3T} + v_2 - v_1 e^{-g_3T} - a_3\left(2e^{2g_1T} - 2e^{g_1T} \times \cos g_2 T\right) + \\ + a_4\left(-e^{2g_1T} e^{-g_3T} - e^{2g_1T} - 2e^{g_1T} \times e^{-g_3T}\cos g_2 T\right) + \\ + a_5\left(e^{2g_1T} + 2e^{g^1T} e^{-g_3T}\cos g_2 T\right) \end{bmatrix};$$

$$d_{1,2} = \frac{1}{T}\begin{bmatrix} -2v_2 - a_4 + e^{-g_3T}(a_1 - v_2 - a_3 - a_5) - a_1 e^{2g_1T} + \\ + 2e^{g_1T}\cos g_2 T(-a + a_3 - a_5) + 2e^{g_1T} e^{g_3T}\cos g_2 T \cdot a_4 \end{bmatrix}; (26)$$

$$d_{1,3} = \frac{1}{T}\left[a_1\left(2e^{g_1 T}\cos g_2 T - 1\right) + v_2 + a_3\left(e^{-g_3 T} - 1\right) + a_5\right]. \qquad (27)$$

TABLE I

IM COEFFICIENTS K=0

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_j$ | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 |
| $d0_{0j}$ | 0.078 | 0.091 | 0.082 | 0.07 | 0.058 | 0.048 | 0.041 | 0.035 | 0.03 | 0.027 | [illegible] |
| $d0_{1j}$ | -0.2 | -0.295 | -0.334 | -0.34 | -0.328 | -0.304 | -0.272 | -0.235 | -0.195 | -0.153 | [illegible] |
| $d0_{2j}$ | 0.128 | 0.242 | 0.345 | 0.442 | 0.533 | 0.619 | 0.7 | 0.775 | 0.845 | 0.908 | [illegible] |

TABLE II

IM COEFFICIENTS K=1

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $T_j$ | 0.05 | 0.1 | 0.15 | 0.2 | 0.25 | 0.3 | 0.35 | 0.4 | 0.45 | 0.5 | 0.55 |
| $d1_{0j}$ | 0.038 | 0.041 | 0.035 | 0.027 | 0.02 | 0.015 | 0.011 | $8.284\cdot10^{-3}$ | $6.343\cdot10^{-3}$ | $4.936\cdot10^{-3}$ | $3.9\cdot10^{-3}$ |
| $d1_{1j}$ | -0.057 | -0.09 | -0.107 | -0.113 | -0.113 | -0.108 | -0.102 | -0.093 | -0.084 | -0.074 | -0.064 |
| $d1_{2j}$ | -0.04 | -0.04 | -0.016 | 0.023 | 0.071 | 0.124 | 0.179 | 0.235 | 0.289 | 0.341 | 0.39 |
| $d1_{3j}$ | 0.066 | 0.126 | 0.182 | 0.235 | 0.286 | 0.334 | 0.381 | 0.425 | 0.468 | 0.509 | 0.548 |

Compiled graphs in Fig. 1 show that if the quantization period T is reduced, discrete poles tend to 1, and the information field (content) of the simulation model decreases. In this case, linear dependence increases and reverse matrix $U^{-1}$ operation is not feasible between columns of matrix U (2).

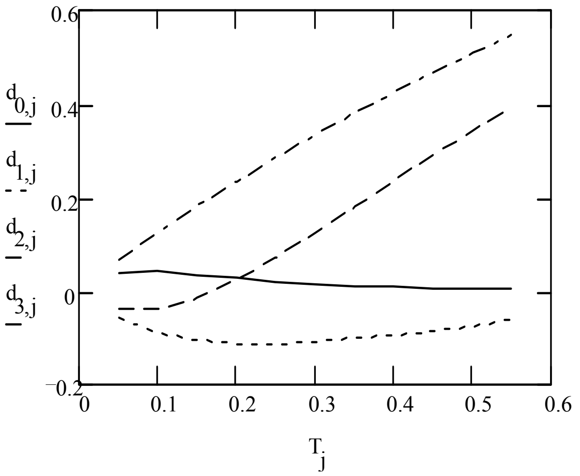


Fig. 1. Operator $D^{(1)}{}_{m}(z)$ coefficients.

## IV. ANALYSIS OF DIAGNOSING THE ALGORITHM IMPLEMENTATION OPPORTUNITY

Sometimes, it is necessary to carry out a thorough diagnosis by obtaining numerical value of the parameter $\hat{W}(p)$ vector based on (5), where the solution of (2) is used. Let us consider the automatic diagnosis system based on the following model:

$$\Phi z(T) * \{x(t); y(t); U\} \Rightarrow \hat{D}_{MU}(z) \Rightarrow \hat{W}_U(p) \qquad (28)$$

where subscript U indicates that the output data for diagnosis were received with empirical errors when solving the system of equations (2). It has already been mentioned that the matrix can increase the background noise. Therefore, the incorrect application of model (28) may decrease the diagnostic reliability of the results obtained. In accordance with formulas (3) and (6), the operator $\hat{D}_M(z)$ is formed as a function:

$$D_{M,U} = \left[\frac{R_{M,U}(z)}{B_{MU}(z)}\right]. \qquad (29)$$

These polynomials are formed separately on the basis of the vector coefficients $\overline{\alpha}$ and $\overline{\xi}$, in the vector $\overline{\theta}_U$ composition. Object transformation function $\hat{W}_U(p)$ score is obtainable depicting discrete operator $\hat{D}_M(z)$ in analogue form. From equations (5) - (8) we get that this representation is feasible using the inverse operation for Z-modification, denoted as operator $\Phi z^{-1}(T)$. Therefore, for model (28) evaluation it is necessary to analyze the nature of computing operations, in order to evaluate inverse operator

$$\Phi z^{-1}(T) * \hat{\overline{\theta}} \Rightarrow \hat{W}(p). \qquad (30)$$

These operations are similar to (8) - (15), but operations are taking place in reverse. We must start by the characterizing polynomial (29) common denominator $B_{MU}(z)$ root definition. This analysis has shown that the individual root obtaining operation is imperative for any standardized computing algorithm. It may be impossible in the case of compressed information (impossible to distinguish between calculated roots). At baseline, analog poles for operator $W(p)$ were well differentiated, as they were on the left plane of complex numbers and the distance between them was large enough.

However, according to (15), these roots moved to the right complex plane in a restricted area at reduced distances. If the root finding operation cannot be carried out accurately, the next operation – the distribution of the vector $\hat{D}_M(z)$ sum of elementary fractions (9), (10) cannot be carried out. It is concluded from the formula:

$$V_i = \frac{R_M(z = \beta_i)}{\prod_{i=1 (j\neq i)}^{n} h_{ij}}; \quad h_{ij} = \left(\beta_j - \beta_i\right) \qquad (31)$$

If for discrete pole splitting the Taylor series is used, then from (31) one obtains:

$$\prod_{i=1\,(i\neq j)}^{n}\left(\beta_j-\beta_i\right)=T^{n-1}\cdot\prod_{i=1\,(i\neq j)}^{n}\left(a_j-a_i\right) \tag{32}$$

$$V_J\geq\frac{1}{T^{n-1}}\left|\frac{R_U(z=a_i)}{\prod_{i=1\,(i\neq j)}^{n}\left(a_j-a_i\right)}\right| \tag{33}$$

Further it is necessary to establish $D_{M,U}$ (29) as a sum

$$D_M(z)\Rightarrow\sum_{i=1}^{n}\frac{V_i\cdot z}{z-\beta_i} \tag{34}$$

and to transform the summarized parts by using the inverse operation of formula (11).

They are nonlinear discrete-pole $\beta_i$ operations for turning into analogues $\vartheta_i$

$$\left[\beta_i=\exp(-T\vartheta_i)\right]\Rightarrow\left[\vartheta_i=\frac{\ln(\beta_i)}{T}\right] \tag{35}$$

Using the formula

$$\Phi z^{-1}(T)*\left[\frac{V_i\cdot z}{z-\beta_i}\right]\Rightarrow\frac{V_i}{p-\vartheta_i} \tag{36}$$

expression (30) turns in the form:

$$\Phi z^{-1}(T)*D_M(z)\Rightarrow\left\{\hat{W}(p)=\sum_{i=1}^{n}\frac{V_i}{p-\vartheta_i}\right\}\Rightarrow\frac{\hat{R}_W(p)}{\hat{Q}_W(p)} \tag{37}$$

Therefore, the model (28) computing operations are mathematically incorrect. Abstract vector $\overline{\theta}_U$ (2) cannot modify parameters that characterize the physical state of the object.

## V. Conclusions

Traditional identification models based on conventional arithmetic equation solutions for the object of signal measurements cannot be used for computerized automatic control and diagnostic systems.

The arithmetic equation vectors $\overline{\theta}$ are abstract. Their relation to physical parameters can be quantified in the analytical relationship. However, there are mathematically incorrect operations and results are not transparent.

The operational computer software of automatic control and diagnostic systems of vector $\overline{\theta}$ decrypting and development technical objects is doomed to failure. It also refers to the use of stochastic identification models.

For the in-depth diagnosis of the object technical state, new methods must be used.

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