

Readjusting for Digital Transformation: A Primary Mathematics Framework

Ildze Čakāne
University of Latvia
Riga, Latvia
ildze.cakane@lu.lv

Ilze France
University of Latvia
Riga, Latvia
ilze.france@lu.lv

Astrida Cirule
Concordia University Chicago
River Forest, IL USA
Univerisyt of Latvia
Riga, Latvia
astrida.cirule@lu.lv

Abstract— The increasing digitalization of education necessitates reconsidering primary mathematics instruction. This study explores the integration of blended learning in Latvian primary mathematics classrooms, addressing the evolving roles of students, teachers, and curricula in the digital age. Using a mixed-methods approach, we analyzed 60 mathematics lessons from 2022 to 2024 to assess student learning opportunities for blended learning. The findings indicate that while digital technologies are present, their pedagogical use remains limited. No observed lessons fully implemented blended learning, though some teaching profiles demonstrated elements of self-regulated and deep learning. Based on these findings and a review of the literature, we propose a conceptual framework for content and process integration for mathematics teaching in the digital age. The framework emphasizes the significant role of the digital dimension complementing the traditional didactic triangle of learner, teacher, and curriculum. This highlights and influences how we view self-regulation, communication, motivation, and process-content integration. Our study highlights the need for curricular adjustments and targeted professional development to better align teaching practices with the realities of digital transformation, ensuring students develop the necessary competencies for the future.

Keywords— *Blended learning, digital transformation, mathematics education, primary education.*

I. INTRODUCTION

Workplace and education landscapes are changing due to digitalization and the development of technologies. The later years of formal education cannot be imagined without including blended learning to some extent [1]. This asks for a set of relevant skills and not just digital competencies [2]. Change is not just in the form of learning, the actual content needs to be adjusted for the changing skillset needed to ensure student success. When thinking about teaching in the present day, it should not be forgotten that childhood and adolescence have changed too - constantly surrounded

by screens and ongoing entertainment. Student experiences before and during formal education are vastly different than those of previous generations and this should be taken into consideration when thinking about their teaching and learning. The change does not stop here. The role of the teacher is rapidly changing as well due to the development of generative artificial intelligence and individual virtual tutors being accessible to students. All of the above-mentioned factors are highly influenced by the circumstances of the country in which the person lives and other contextual (e.g., socioeconomic, cultural) aspects.

"In recent years, the internationalisation and globalization of the economy, universality of technological development and related needs for new skills and knowledge play the role of strong motivations for curriculum reforms"[3]. Blended learning, particularly through flipped learning, rethinks the organization of the learning landscape by moving direct instruction to individual learning spaces and transforming group spaces into interactive environments. This approach, supported by digital technologies, enhances student engagement, interaction, and understanding, though it can also present challenges due to increased workload and unfamiliarity with the method [4].

The design of an effective mathematics curriculum is influenced by key educational theories that focus on how students learn, process, and apply mathematical concepts. These theories guide the selection of content, teaching strategies, and assessment practices to promote deep understanding, critical thinking, and problem-solving skills.

In this research paper, we aim to explore what these changes mean for primary mathematics education in Latvia and conceptualize the involved aspects within a theoretical framework.

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RQ1 What opportunities do students have for blended learning in primary mathematics in Latvia?

RQ2 How to develop a primary mathematics framework in the midst of digital transformation?

A. Digital transformation and blended learning in primary mathematics

Students, who attend school at present day are “digital natives” who have transferred many aspects of their lives to digital technologies and have intuitive navigation skills on many digital devices and applications. Their skills and experiences are vastly different from previous generations of students [5]. Teachers and students have a large gap in experiences and values and it affects teacher understanding of their students and their needs. Students have a different starting point when entering mathematics classrooms. For example, the experience of physical money is replaced by credit cards, analogue watches with digital watches, playing with blocks and roleplay games with video games and videos (sometimes educational).

Teachers must therefore balance traditional mathematical skills with newer competencies that emphasize real-world applications and digital fluency. As digital tools become more accessible, curricula are shifting from repetitive manual calculations to tasks that engage students in mathematical reasoning and problem-solving using technology [6].

We agree with the idea that there is no didactic triangle anymore, but rather a didactic pyramid (Dasari et al.2023), where there are four vertices – student, teacher, curriculum and digital technologies. The digital technologies are not solely a different platform for learning or another way to represent the learning content, but they provide possibilities to gather precise data for teachers, receive immediate individualized feedback for students, boost engagement and we cannot forget the above-mentioned experiences and skills of digital natives. Digitally literate students can access, create, and communicate information, adapt to changing technologies, and use information and communications technologies ethically and responsibly. This competency should be strongly emphasized in curricula [6].

With digital technologies entering the classroom so does blended learning and that can be beneficial for student outcomes since “the use of blended learning has a major effect on students' mathematical abilities when compared to traditional learning”[7], if the teacher has appropriate knowledge, skills and tools.

B. Curriculum changes

Designing a mathematics curriculum in the digital age requires a careful balance of content and process. Our content and process integration framework was designed to emphasise the changes necessary due to digital transformation and based on the new curriculum in Latvia in order to help teachers adapt and implement the changes.

When looking at other countries' successful implementation of needed changes, the curriculum must emphasize the importance of problem-solving, reasoning,

and real-world applications. It must ensure that students not only learn mathematical procedures but also understand their practical relevance. The Australian framework's [8] organization around key content and proficiency strands ensures a holistic and practical understanding of mathematics, integrating various domains such as numbers and algebra, measurement and geometry, and statistics and probability. By focusing on understanding, fluency, problem-solving, and reasoning, the curriculum supports the development of comprehensive mathematical skills. Adopting a spiral approach, as seen in the Singaporean curriculum [9], is crucial for revisiting and deepening the understanding of key concepts over time. The Concrete-Pictorial-Abstract approach further enhances this by helping students develop a strong conceptual understanding before moving on to abstract procedures. Promoting inquiry-based learning is another vital aspect of an effective mathematics curriculum. Encouraging exploration, questioning, and problem-solving fosters critical thinking and deeper understanding. This student-centred approach aligns with constructivist principles, where learners actively engage with mathematical problems and discover solutions through guided inquiry [10]. Such a comprehensive approach ensures that students develop deep mathematical understanding, critical thinking skills, and the ability to apply mathematics in a variety of contexts, preparing them for success in the digital age.

The Programme for International Student Assessment (PISA) stresses the role of mathematical literacy as a central goal in school mathematics education because it improves the life opportunities for students, and justifies why mathematics is essential to describe, explain and predict the world [11]. The curriculum content and methods for introducing revised curricula cannot be directly transferred from one country to another. They can vary based on cultural and historical backgrounds, traditions, as well as political decision-making processes [2]. It should be noted that digital competency frameworks are general and not subject-specific, but they are also likely to influence subject-matter content [4].

An Organisation for Economic Co-operation and Development (OECD) study [6] highlights gaps between curriculum content and real-world applications, particularly in mathematics. The slow adaptation of curricula from routine calculations to problem-solving and real-world applications, combined with traditional rote learning methods, may not adequately prepare students for higher-order skills, leading to declining performance.

C. Key learning processes in the digital age and blended learning

There is a shift in mathematics education from individual knowledge acquisition to collaborative and public practices that co-construct knowledge. Digital resources enhance this shift by enabling communication and collaboration over long distances and asynchronously, placing these elements at the center of modern pedagogies [4]. For students to thrive in a blended learning environment, their teachers must think about student self-regulated learning, communication and motivation. These

key aspects besides digital fluency are becoming more important for learning in the digital age [12].

1) Self-regulated learning

Skills related to self-directed learning or self-regulated learning are known to be the skills that each successful learner needs to obtain [13]. These skills are necessary not only for continuous learning needed in many workplace environments but for the future of student's academic learning. Inescapably it will be in some way blended learning, where parts of the learning happen in a digital environment and at least somewhat independently. The learner must regulate his behaviour and motivation to start a task, keep his attention on it and reflect on the process and results. Research shows [14] that for students to be able to thrive in blended learning environments they need self-regulated skills. These skills help not only for blended learning but are beneficial for overall mathematics performance for primary students [14]. We will be using the term self-regulated learning for the primary school context, since self-directed learning is associated with the learner's full responsibility for all of the learning process, determining goals and all the steps towards it [15]. Still, self-regulated learning is on a reduced scale, where the learner's responsibility is around particular tasks and projects [15]. We consider the term self-regulated more appropriate since in primary school the learning goals are predetermined by the curriculum, program and teacher and the tasks and learning process are designed by a teacher (even if students have opportunities to express opinions when defining the goals or choosing tasks, they do not have a say in what they have to learn when finishing each step in their formal education) and the responsibility for his learning experience lies on the teacher [16]. Self-regulated skills are related to (meta)cognitive strategy use, motivational, emotional and behavioural regulation of oneself [17]. These skills are used throughout all the phases of self-regulated learning [18]– the forethought phase, the performance phase and the self-reflection phase.

Self-regulated learning and student agency have also yielded changes in the pedagogies to make use of the affordances of digital resources to foster these new educational ideals.

a) Metacognition

We would like to stress the importance of metacognitive awareness and skills when thinking about mathematics education and blended learning. Metacognition involves the conscious awareness and oversight of one's thought processes, including recognizing how and why you think or act in certain ways, as well as making deliberate choices to modify or adjust them [19]. This reflective practice helps students develop more effective learning strategies, connect mathematical concepts, and improve their overall performance in mathematics [20]. This can be taught by explicitly teaching cognitive and metacognitive strategies, modelling the use of metacognitive skills (e.g., "thinking aloud") and investigating the thought processes of others [21], [22]. Integrating metacognitive strategies into blended learning

models significantly improves students' ability to understand mathematical concepts and manage their learning processes [23].

2) Communication

Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically; analyze their peer's mathematical ideas and strategies [24]. It helps students develop their understanding of mathematics and enhance their mathematical reasoning [25]. The skill evolves if there is an engaging, inclusive classroom discussion leading to deep thinking and diverse and purposeful representations and explicit transitions are used [24].

a) Classroom discourse

The typical approach to questioning in elementary mathematics classrooms in Latvia often involves only recall, where teachers expect a single correct answer without delving into students' reasoning [26]. Even when students are asked to explain their solutions, teachers usually anticipate a single correct method. This approach confirms individual students' knowledge but does not facilitate deep learning.

During the learning or comprehension phase of a lesson discussions should be structured to allow students to learn, explain, reason, and deepen their understanding. Productive classroom discussions involve various conversational tools or talk moves that encourage students to articulate their ideas, listen to others, recognize different approaches, and evaluate them critically. These strategies include [27]:

Revoicing: the teacher repeats or paraphrases the student's response and asks for confirmation or clarification. This ensures the understanding of the student's idea and makes it accessible to others.

Repeating: asking students to restate another's reasoning in their own words. This helps reinforce understanding and validates peer contributions.

Reasoning: using student ideas to continue the discussion, by asking students to reason about other reasoning. Asking students to evaluate or justify ideas.

Adding on: Prompting students to extend their own or others' thoughts.

Wait time ("Thinking time"): Allowing sufficient time for students to process questions and formulate responses is critical. Research [28] shows that teachers typically wait only about one second for a response. However, when wait time exceeds three seconds, student engagement, response quality, and cognitive depth improve significantly.

Pair or small group discussions (turn and talk): Ask students to exchange ideas with a partner or small group before sharing with the whole class. This move gives students the opportunity to formulate their ideas, ascertain that they are thinking in the right direction or arrive at a

solution together. It gives each student the opportunity to use mathematical language.

Additionally, it is important for teachers to evaluate the questions they pose to make sure they are not solely relying on those that require only basic cognitive skills. They should avoid prematurely giving students answers, as this can be demotivating.

b) Representations

Mathematical understanding deepens when students engage with multiple representations of concepts, including arithmetic operations. Using various representations expands comprehension, helping students determine when and how to apply mathematical operations and interpret results [29]. Effective mathematics instruction connects representations to reinforce conceptual understanding and problem-solving skills [30].

A mathematical representation is a mental or tangible construct that describes a concept and its relationships. Experiencing a concept across different contexts strengthens understanding [31]. Representations serve several roles in learning [30]: establishing a foundation for mathematical understanding, clarifying mathematical contexts, identifying relationships between concepts, and enabling communication of understanding and reasoning.

Representations illustrate mathematical ideas. There are five representations (static pictures, written symbols, spoken language, real scripts, and manipulative models) and their interconnections [32], or six, if we count digital representations as a separate category since it provides the possibility of “moving pictures” [33]. Crucially, students must develop the ability to transition between representations, selecting the most effective form for problem-solving. Teachers in Latvia struggle to incorporate all of the representation in their teaching [34].

3) Motivation

Teacher beliefs about their students’ motivation and what affects it (e.g. whether the teacher can influence it or it is all up to the student) are shaped by a variety of factors [35], [36]. First, their knowledge about what motivates students (from teacher education) and learning (e.g. beliefs that student achievement is solely or mostly dependent on an innate ability[37]). Teachers’ philosophical views on motivation and mathematics learning (behaviourism vs autonomy, supportive teaching, direct instructions and memorization vs learning through problem-solving) are another factor. In addition, personal experiences related to mathematics learning and motivation influence teacher beliefs. Teacher beliefs have been shown to affect student outcomes [38].

For exploring motivation in the classroom and the teacher’s role in it, we use self-regulation theory because of its broad research and straightforwardness.

II. MATERIALS AND METHODS

A mixed research design was used for this research.

The study sample consists of 60 primary mathematics lessons (grades 1-6). The lessons were observed, transcribed and rated across multiple criteria on a scale from 0 to 4 between the years 2022 and 2024 by experienced and specially trained primary STEM teaching experts. No information about teacher and student demographic characteristics is available to the authors. For the present research, the authors obtained the lesson transcripts and ratings from a database. A rubric “Teaching and Learning Assessment to Support the Teaching of Student 21st-Century Skills”[39] was used to determine teaching performance levels by comparing theoretically described preferable practices and the observed ones.

TABLE I. AN EXAMPLE OF A CRITERION “CLASSROOM DISCOURSE” WITH LEVEL DESCRIPTIONS

Level	Description
0 (Not observed)	The teacher introduces information and explains concepts. If questions are asked, they are closed. Students mainly listen, observe, and take notes.
1 (Beginner)	The teacher’s monologue dominates. However, some elements of discourse can be observed – the teacher asks questions but expects specific answers. Questions are only used for checking knowledge. If students fail to provide prompt expected answers, the teacher answers them him/herself. Students answer particular questions and/or ask very few questions.
2 (Developing)	Alongside explaining and asking questions, the teacher creates a situation where students have to pose questions about the content of the lesson. If needed, the teacher instructs students on how to form questions and how to engage in conversation. The teacher tries to provide enough time for students to think over their answers; however, the time is often insufficient. Students feel free to ask questions and engage in conversation.
3 (Proficient)	The teacher asks diverse questions, including those where there is no single right answer, and asks for students’ thoughts and reasoning. The teacher uses questioning and different student responses to extend the discourse either frontally or in groups (pairs). If needed, the teacher rephrases student questions. The teacher gives students enough time to think during the discussion. Students have a clear understanding of the procedure of classroom discourse – how to ask and how to answer. Students ask diverse questions.
4 (Expert)	With appropriate questions, the teacher creates situations to engage students and deepen their thinking. Students ask diverse questions, initiate and lead discussions, and are active and fully engaged in thinking.

The rubric consists of 17 criteria, in each criterion, teaching practices are described in 5 levels. The teaching practice is rated at level 0 if the respective criterion is not observed; and level 4 if the teaching practice represents exceptional practice. An example of the criterion “classroom discourse” with level descriptions is provided in Table 1.

The lesson rating was done by the same experts, who observed and transcribed the lessons; a substantial

interrater agreement had been achieved before the experts rated the lessons individually.

For determining typical student opportunities (and differences) for blended learning in mathematics classrooms a profiling approach was used. This approach identifies distinct groups or profiles of teaching that exhibit similar patterns of strengths and areas for improvement [40]. The profiles are derived from classroom observational data and help in summarizing and interpreting the teaching quality and professional development needs of teachers. A previously developed decision tree with respective categories and criteria (Table 2) for the determination of primary STEM teaching profiles was used [41].

TABLE II. CATEGORIES THAT CONCEPTUALIZE BLENDED LEARNING IN STEM AND THEIR RESPECTIVE CRITERIA

Categories	Criteria
1. Opportunities to learn with digital technologies	1.1. Meaningful use of digital technologies by students
2. Student self-regulated learning opportunities	2.1. Clear learning objectives
	2.2. Feedback to students
	2.3. Opportunities for metacognition
	2.4. Differentiation and support for learning
3. Student deep learning opportunities	3.1. Curriculum representation
	3.2. Tasks for deep learning
	3.3. Classroom discourse

III. RESULTS AND DISCUSSION

A. RQ1

There were no lessons observed where blended learning was fully implemented or the kind of teaching where all the skills necessary for blended were used. In some 12% of the sample lessons digital technologies were used, but in none of the lessons were they used for learning. Nevertheless, the profiles show how the student learning opportunities prepare them for later blended or independent learning (see Table 3).

There were 4 distinct teaching profiles. Teaching that corresponds with **profile 1** provides students with some self-regulated and deep learning opportunities. **Profile 2** teaching provides students with some self-regulated but very limited deep learning opportunities. **Profile 3** teaching provides students with some deep learning but very limited self-regulated learning opportunities. **Profile 4** teaching provides students with very limited self-regulated learning and deep learning opportunities.

TABLE III. AVERAGE PERFORMANCE LEVELS ACROSS THE SELECTED CATEGORIES AND CRITERIA CORRESPONDING TO THE FOUR OBSERVED PRIMARY MATHEMATICS TEACHING PROFILES (N=60)

Profile	No. of lessons	Cat. 1	Cat. 2				Cat.3		
		1.1.	2.1.	2.2.	2.3.	2.4.	3.1.	3.2.	3.3.
1	19 32%	0,2	2,0	1,8	0,7	0,9	1,8	1,3	1,5
2	8 13%	0,1	1,4	1,4	0,4	0,5	0,8	0,5	0,8
3	6 10%	0,0	0,7	0,8	0,3	0,0	1,3	1,3	1,2

4	27 45%	0,1	0,6	0,9	0,2	0,3	0,7	0,6	0,7
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When looking at the categories (Fig.1), students had more opportunities for deep learning than self-regulation. Within the categories, students often received clear, specific, measurable and achievable learning objectives and feedback concerning the planned learning objective or the learning process, but very rarely received different tasks or other ways to learn according to their abilities or used metacognitive strategies (Cat.2.). Within the 3rd category – all criteria were similarly observed overall, with performance a little higher in content representation and work with student perception. Learning tasks were mostly reproductive and required low cognitive skills, and discourse was mostly one-way - where the teacher does a lot of talking and asks questions primarily for recall.

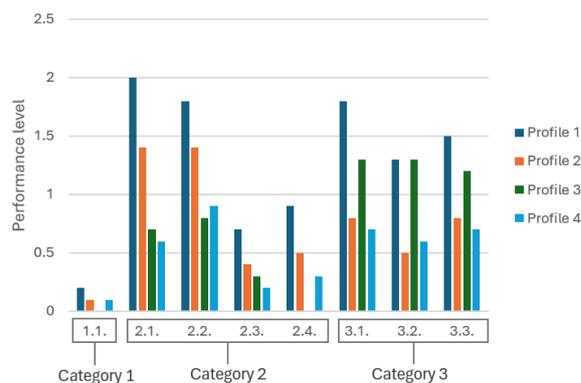


Fig. 1. Bar chart of the average performance for the teaching profiles in each criterion.

B. RQ2

Based on literature analysis and taking into consideration the given context a framework for content and process integration for mathematics teaching in the digital age was created.

As previously mentioned, we consider the students' experience in the digital world as a crucial element which influences the primary mathematics curriculum. Additionally, we assert that the curriculum should set objectives for students who are well-versed in the digital world, regularly accessing information, communicating, and learning through digital technologies. Thus, the framework includes (see Fig. 2) digital technologies as a separate dimension that intertwines with other framework elements and encompasses digital competence and the ability to learn via digital technologies. The introduction of digital resources does not only affect the mathematics and related learning goals but also influences students' interaction with mathematics. Besides digital technologies, other dimensions of learning consist of self-regulation, communication and motivation.

The framework also includes five content strands/categories: numbers, measurement, geometry and spatial thinking, data and uncertainty and relationships. The Framework defines content knowledge as the

understanding of mathematical concepts, procedures, facts, and tools necessary to describe, explain, and predict phenomena, ensuring that students can apply mathematics in various real-world contexts. Thus, the four mathematical proficiency strands are defined: conceptual understanding, procedural understanding and fluency, mathematical reasoning, problem-solving and mathematical modelling. Students should have opportunities to apply their

mathematical problem-solving and reasoning skills to a variety of challenges. Through mathematical modelling, they learn to navigate ambiguity, make connections, select and apply appropriate mathematics concepts and skills, identify assumptions, reflect on the solutions to real-world problems, and make informed decisions.

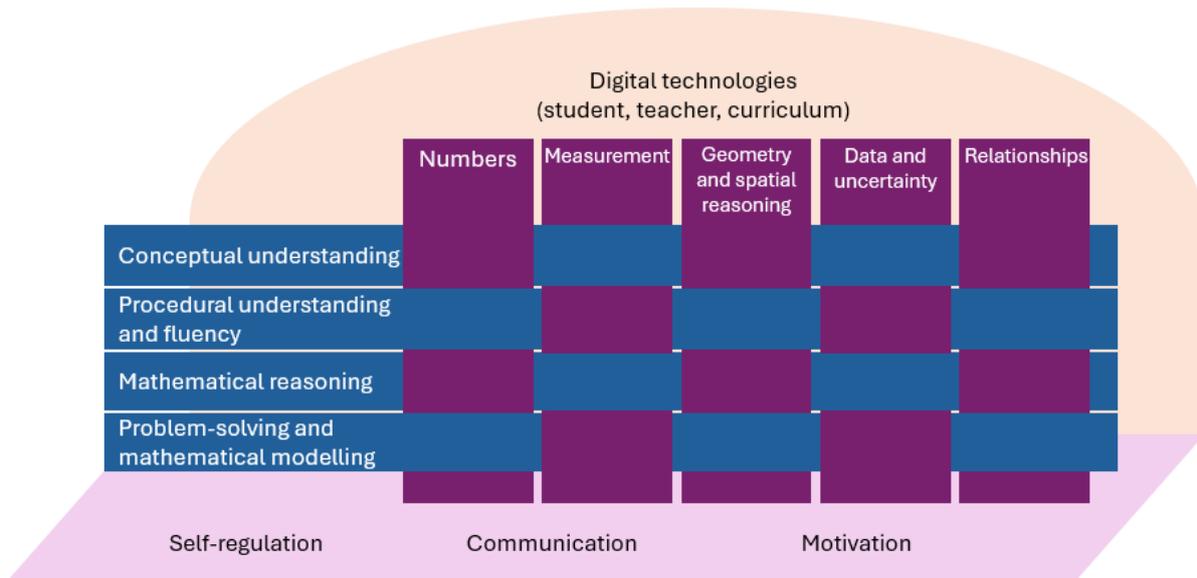


Fig. 2. Content and process integration framework for mathematics teaching in the digital age.

The study, similar to Rezat and Geiger, suggests that while digital resources have the potential to make the sequencing of mathematical content more flexible and adaptable to individual learners, changes to school curricula and school practices have been mostly superficial so far. Providing primary school students with the appropriate tools and language may lead to more personalized learning environments in the future [4].

There are several issues concerning digital transformation that we did not look into in this paper, but that require discussion. While the virtual representation of mathematical objects using the advantages of digital technologies allows for operations with these objects that would not be possible in real life, the question of how students' operations with virtual instead of physical objects influence their learning of mathematics is unresolved. This requires further research since constructivist learning theories have always argued that conceptual development in mathematics is grounded in sensorimotor experiences in the real world and a result of ongoing abstraction [4]. The impact of artificial intelligence systems will significantly accelerate changes in education, as their increasing availability suggests that such transformations are imminent. While digital texts and blended learning offer benefits like easy updating and accessibility, their impact on teachers and learners in curriculum reform is less understood. These resources have the potential to support curriculum communication and adaptation, but their

effectiveness in pedagogical discourse remains complex and context-dependent [42].

Further research should continue on how continuously monitoring students' progress using technology teachers can identify areas where students are struggling and provide targeted interventions to address misconceptions or gaps in knowledge. Effective assessment practices are essential for both measuring student progress and refining the curriculum to better meet the diverse needs of learners.

IV. CONCLUSIONS

This study highlights the limited yet emerging integration of blended learning in Latvian primary mathematics classrooms. While digital technologies are present, their pedagogical use remains underdeveloped, with no fully implemented blended learning lessons observed. Our findings reveal that teaching practices fall into distinct profiles, demonstrating differences in providing self-regulated and deep learning opportunities that are crucial for learning in the digital environment. There is a predominant reliance on teacher-centred discourse and low-cognitive-demand tasks.

To bridge this gap, we propose a content and process integration framework that recognizes the digital dimension as an essential component of mathematics education today. By accepting the expansion of the traditional didactic triangle into a didactic pyramid (including digital technologies alongside the learner,

teacher, and curriculum) we emphasize the necessity of aligning teaching practices with student experiences. The framework underscores the importance of digital literacy, self-regulation, communication, and motivation, ensuring that students develop the competencies required for a technology-driven learning landscape.

Ultimately, for blended learning to become a meaningful component of primary mathematics education, systemic curricular adjustments and targeted professional development initiatives are essential.

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REFERENCES

- [1] C. Hague, “Fostering higher-order thinking skills online in higher education,” Feb. 2024. doi: 10.1787/84f7756a-en.
- [2] Future of Jobs Report, World Economic Forum, 2025. [Online]. Available: www.weforum.org. [Accessed: Feb. 23, 2025].
- [3] Y. Shimizu and R. Vithal, “School Mathematics Curriculum Reforms: Widespread Practice But Under-Researched in Mathematics Education,” in *Mathematics Curriculum Reforms Around the World*, New ICMI Study Series, Y. Shimizu and R. Vithal, Eds., 2023, pp. 3–21. doi: 10.1007/978-3-031-13548-4_1.
- [4] S. Rezat and V. Geiger, “The Role of Digital Technologies in Transforming Student Learning Landscapes,” in *Handbook of Digital Resources in Mathematics Education*, B. Pepin, G. Gueudet, and J. Chopin, Eds., Springer, Cham, 2024, pp. 1–22. doi: 10.1007/978-3-030-95060-6_21-1.
- [5] E. Ann Rose and M. R. Thomas, “Generation Alpha and Learning Ecosystems,” in *Preparing Students for the Future Educational Paradigm*, IGI GLOBAL, 2024, pp. 19–46. doi: 10.4018/979-8-3693-1536-1.CH002.
- [6] *An Evolution of Mathematics Curriculum: Where It Was, Where It Stands and Where It Is Going*, OECD, 2024. doi: 10.1787/0ffd89d0-en.
- [7] D. N. Mawardi, C. A. Budiningsih, and Sugiman, “Blended Learning Effect on Mathematical Skills: A Meta-Analysis Study,” *Ingenierie des Systemes d’Information*, vol. 28, no. 1, pp. 197–204, Feb. 2023, doi: 10.18280/isi.280122.
- [8] I. V. S. Mullis, M. O. Martin, S. Goh, and K. Cotter, “TIMSS 2015 Encyclopedia: Education Policy and Curriculum in Mathematics and Science,” 2016.
- [9] *Mathematics Syllabus: Primary One to Six*, Ministry of Education, Singapore, 2012. [Online]. Available: https://www.moe.gov.sg/-/media/files/primary/mathematics_syllabus_primary_1_to_6.pdf. [Accessed: Feb. 23, 2025].
- [10] M. Vasuki, C. Mbonigaba, and K. Dinesh, “Curriculum Design in Mathematics: A Review of Effective Frameworks and Models,” *Journal of Engineering, Scientific Research and Applications*, vol. 2, no. 2, pp. 207–222, 2016.
- [11] Organisation for Economic Co-operation and Development, *PISA 2022 Assessment and Analytical Framework*. OECD, 2023. doi: 10.1787/dfef0bf9c-en.
- [12] K. Greitāns, Ģ. Burgmanis, and D. Namsone, “Towards Digital Transformation in Primary Science: Typology of Blended Learning Models,” *CSEdu*, to be published, 2025.
- [13] M. O. Yurdal and Ç. Toraman, “Self-Directed Learning, Academic Achievement and Motivation: A Meta-Analytical Study,” *Alberta Journal of Educational Research*, vol. 69, no. 2, pp. 233–253, Jun. 2023, doi: 10.55016/OJS/AJER.V69I2.75098.
- [14] Z. Xu, Y. Zhao, B. Zhang, J. Liew, and A. Kogut, “A meta-analysis of the efficacy of self-regulated learning interventions on academic achievement in online and blended environments in K-12 and higher education,” *Behaviour and Information Technology*, vol. 42, no. 16, pp. 2911–2931, 2023, doi: 10.1080/0144929X.2022.2151935.
- [15] K. Saks and Å. Leijen, “Distinguishing Self-directed and Self-regulated Learning and Measuring them in the E-learning Context,” *Procedia Soc Behav Sci*, vol. 112, pp. 190–198, Feb. 2014, doi: 10.1016/j.sbspro.2014.01.1155.
- [16] A. Voskamp, E. Kuiper, and M. Volman, “Teaching practices for self-directed and self-regulated learning: case studies in Dutch innovative secondary schools,” *Educ Stud*, vol. 48, no. 6, pp. 772–789, 2022, doi: 10.1080/03055698.2020.1814699.
- [17] M. Boekaerts, “Emotions, emotion regulation, and self-regulation of learning,” in *Handbook of self-regulation of learning and performance*, B. Zimmerman and D. Schunk, Eds., Routledge/Taylor & Francis Group, 2011, pp. 408–425.
- [18] B. J. Zimmerman, “Becoming a self-regulated learner: An overview,” 2002, Ohio State University Press. doi: 10.1207/s15430421tip4102_2.
- [19] J. A. Van De Walle, K. S. Karp, J. M. Bay-Williams, J. Wray, and E. T. Brown, “Elementary and Middle School Mathematics Teaching Developmentally.”
- [20] E. R. Fyfe, C. Byers, and L. J. Nelson, “The Benefits of a Metacognitive Lesson on Children’s Understanding of Mathematical Equivalence, Arithmetic, and Place Value,” *J Educ Psychol*, vol. 114, no. 6, pp. 1292–1306, 2022, doi: 10.1037/edu0000715.
- [21] *Metacognition and Self-Regulated Learning: Guidance Report*, Education Endowment Foundation, 2nd ed. [Online]. Available: <https://educationendowmentfoundation.org.uk/education-evidence/guidance-reports/metacognition>. [Accessed: Mar. 30, 2025].
- [22] B. De Jager, M. Jansen, and G. Reezeit, “The development of metacognition in primary school learning environments,” *School Effectiveness and School Improvement*, vol. 16, no. 2, pp. 179–196, Jun. 2005, doi: 10.1080/09243450500114181.
- [23] A. Dardiri and D. Siswoyo, “The effect of blended learning model with metacognitive strategy on mathematics ability and self-regulated learning of elementary school students,” *Journal of Positive Psychology and Wellbeing*. [Online]. Available: <http://journalppw.com>.
- [24] N. Rohid and R. Danu Rusmawati, “Students’ Mathematical Communication Skills (MCS) in Solving Mathematics Problems: A Case in Indonesian Context,” *Anatolian Journal of Education*, vol. 4, no. 2, pp. 19–30, 2019, doi: 10.29333/aje.2019.423a.
- [25] C. Tobey, *The Role of Discourse in Inclusive and Equitable Mathematics Learning and Teaching* [Online]. Available: <https://www.mheducation.com/unitas/school/explore/sites/reveal-math/tobey-discourse-inclusive-equitable-math-white-paper.pdf>. [Accessed: Feb. 23, 2025].
- [26] I. Čakāne, D. Namsone, and I. France, “How students are learning to reason in mathematics lessons,” *Society. Integration. Education. Proceedings of the International Scientific Conference*, vol. 1, pp. 541–552, Jul. 2023, doi: 10.17770/SIE2023VOL1.7087.
- [27] S. H. Chapin, M. C. O’Connor, and N. C. Anderson, *Classroom Discussions: Using Math Talk to Help Students Learn, Grades K-6*. Math Solutions, 2009.
- [28] J. A. Walsch and D. B. Sattes, *Quality Questioning: Research-Based Practice to Engage Every Learner*. Corwin Press, 2016.
- [29] B. Mainali, “Representation in Teaching and Learning Mathematics,” *International Journal of Education in Mathematics, Science and Technology*, vol. 9, no. 1, pp. 1–21, 2021, doi: 10.46328/ijemst.1111.

- [30] D. Huinker and V. Bill, *Taking Action: Implementing Effective Mathematics Teaching Practices in K-Grade 5*. National Council of Teachers of Mathematics, 2017.
- [31] P. N. Tripathi, “Developing mathematical understanding through multiple representations,” *Mathematics Teaching in the Middle School*, 2008.
- [32] R. Lesh, T. R. Post, and M. Behr, “Representations and translations among representations in mathematics learning and problem solving,” in *Problems of Representation in the Teaching and Learning of Mathematics*, C. Janvier, Ed., Lawrence Erlbaum Associates, 1987, pp. 33–40.
- [33] E. L. Johnson, “A New Look at the Representations for Mathematical Concepts: Expanding on Lesh’s Model of Representations of Mathematical Concepts,” *Forum on Public Policy Online*, vol. 2018, no. 1, 2018.
- [34] I. Čakāne and I. France, “An Empirical Study on the Use and Transitions of Representations in Primary Math Lessons,” *Human, Technologies and Quality of Education*, 2024, pp. 269–280, Nov. 2024, doi: 10.22364/HTQE.2024.21.
- [35] D. J. Stipek, K. B. Givvin, J. M. Salmon, and V. L. MacGyvers, “Teachers’ beliefs and practices related to mathematics instruction,” *Teach Teach Educ*, vol. 17, no. 2, pp. 213–226, Feb. 2001, doi: 10.1016/S0742-051X(00)00052-4.
- [36] D. J. Stipek, K. B. Givvin, D. J. Stipek, J. M. Salmon, and V. L. Macgyvers, “In the eyes of the beholder: students’ and teachers’ judgments of students’ motivation,” 2001.
- [37] D. J. Stipek, K. B. Givvin, J. M. Salmon, and V. L. MacGyvers, “Teachers’ beliefs and practices related to mathematics instruction,” *Teach Teach Educ*, vol. 17, no. 2, pp. 213–226, Feb. 2001, doi: 10.1016/S0742-051X(00)00052-4.
- [38] D. J. Stipek, K. B. Givvin, J. M. Salmon, and V. L. MacGyvers, “Teachers’ beliefs and practices related to mathematics instruction,” *Teach Teach Educ*, vol. 17, no. 2, pp. 213–226, Feb. 2001, doi: 10.1016/S0742-051X(00)00052-4.
- [39] D. Bērtule, I. Dudareva, D. Namsone, L. Čakāne, and A. Butkevica, “FRAMEWORK OF TEACHER PERFORMANCE ASSESSMENT TO SUPPORT TEACHING 21ST CENTURY SKILLS,” 13th International Technology, Education and Development Conference, 2019. doi: 10.21125/inted.2019.1410.
- [40] P. F. Halpin and M. J. Kieffer, “Describing Profiles of Instructional Practice: A New Approach to Analyzing Classroom Observation Data,” *Educational Researcher*, vol. 44, no. 5, pp. 263–277, Jun. 2015, doi: 10.3102/0013189X15590804.
- [41] I. Čakāne, K. Greitāns, Ģ. Burgmanis, and D. Namsone, “Towards Blended Learning in Primary STEM in Latvia: Four Teaching Profiles,” *Educ Sci (Basel)*, to be published, 2025.
- [42] J. Golding, “Coherence and Relevance of Materials and Technologies to Support Mathematics Curriculum Reforms,” in *Mathematics Curriculum Reforms Around the World*, Y. Shimizu and R. Vithal, Eds., Springer, Cham, 2023, pp. 173–191. doi: 10.1007/978-3-031-13548-4_12.