

Computer Grafoanalytic Kinematical Analysis for Four-Bar Linkage Mechanism

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Abstract— In this work, a methodology for calculating a class of lever mechanisms is proposed using a current software product for solving algebraic equations. They are adapted in the environment of the theory of mechanisms, where the main kinematic parameters of the articulated four-link mechanism are presented in graphical form. These parameters are linear and angular velocities and linear and angular accelerations. The work is related to the course in general and mechanical engineering at the Technical University - Gabrovo.

Keywords — computer kinematical analyses, four-bar linkage mechanism, Math Lab.

I. INTRODUCTION

The proposed algorithm for the analysis of the most commonly used mechanism allows, in the operating environment of the modern software product Math LAB 2020 for Windows, which has enormous computational capabilities and its graphic images on the monitor allow the reading of information with great accuracy, to accelerate and improve the analysis. The graphic images of the changes in the kinematic quantities (angles of rotation, angular velocities, angular accelerations, etc.) of the links of the mechanism, for one revolution of the input lever, make it possible to easily perform a force analysis of this mechanism. The reliability of the algorithm is established with a detailed solved example.

The analysis is compiled with given: uniform rotation of the leading link 1, the initial angular orientation of this link and the geometric dimensions of all links of the mechanism. The algorithm, as is known [10] and [23] is a finite series of rules, according to which after a finite number of uniquely defined steps a solution to a given problem is obtained, uniquely corresponding to the conditions given in it.

The purpose of the publication is to create a systematic approach for metric synthesis on discrete positions of lever mechanisms using the most commonly used geometric and algebraic methods, through the creation of algorithms and program modules in the Math LAB for Windows environment.

The following main tasks arise from the purpose of the article:

1. Compilation of algorithms for kinematic synthesis of four-link transmission hinge mechanisms using the following methods: the method of symmetries, the method of the maximally contracted evolute, the determinant method, the Alt method, the method using the Freudenstein [1] equation, the method using vector-matrix equations and the analytical-optimization method. The algorithms should be compiled for three, four and five discrete positions.
2. Compilation of algorithms for kinematic analysis of four-link closed kinematic chains, three-link closing parallel chains in the profile of the inserted higher kinematic pair.
3. The values of all sought kinematic parameters should be graphically and analytically reported.

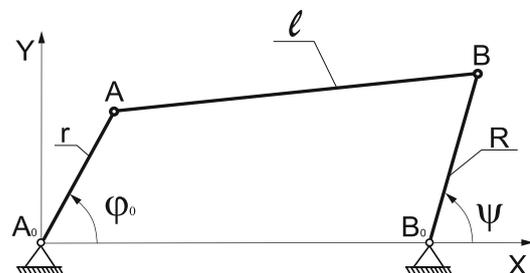


Fig.1 shows a four-bar lever mechanism.

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II. MATERIALS AND METHODS

II.1 Conditions and algorithm of analysis

The algorithm for kinematic analysis is built with the following given conditions: uniform rotation of the leading link, the initial angular orientation of the same link and the geometric dimensions of all links of the mechanism.

The construction of this algorithm is carried out in the following sequence:

1. The input data determined by the analysis conditions (ω , r , l , R , φ_0 , φ_1 , $L=A_0B_0$) are entered.

2. The number of positions n of the mechanism at which the analysis will be performed is determined.

3. The step by which the leading link will rotate when moving from one to an adjacent position is determined.

$$h = \frac{2\pi}{n} \quad (1)$$

4. The number of positions of the mechanism is chosen as a variable parameter $\varphi_k = \varphi_0 + (k-1)h$.

5. Three arrays of values of the angular orientations of the leading link are calculated, each of which is for $n-1$ positions

$$\begin{aligned} \varphi_k &= \varphi_0 + \varphi_1 + (i-3)h, \quad k = -1, 1, 2, \dots, n-2 \\ \varphi_l &= \varphi_0 + \varphi_1 + (i-2)h, \quad l = 1, 2, \dots, n-1 \\ \varphi_m &= \varphi_0 + \varphi_1 + (i-3)h, \quad m = 2, 3, \dots, n \end{aligned} \quad (2)$$

6. Three arrays of coordinates XA and three arrays of coordinates YA of the center of the movable hinge A (connecting the leading link to the reel fork) are determined, at $n-1$ positions of the mechanism.

$$\begin{aligned} X_{Ak} &= r \cdot \cos \varphi_k, \quad k = -1, 1, 2, \dots, n-2 \\ Y_{Ak} &= r \cdot \sin \varphi_k, \\ X_{Al} &= r \cdot \cos \varphi_l, \quad l = 1, 2, \dots, n-1 \\ Y_{Al} &= r \cdot \sin \varphi_l, \\ X_{Am} &= r \cdot \cos \varphi_m, \quad m = 2, 3, \dots, n \\ Y_{Am} &= r \cdot \sin \varphi_m, \end{aligned} \quad (3)$$

7. The six arrays of coordinates of the movable joint B (connecting the crank and the rocker arm) corresponding to the arrays (3) are determined, each of which is respectively for $n-1$ positions of the mechanism:

$$\begin{aligned} X_{Bk} &= L + R \cdot \cos \varphi_k, \quad k = -1, 1, 2, \dots, n-2 \\ Y_{Bk} &= R \cdot \sin \varphi_k, \\ X_{Bl} &= L + R \cdot \cos \varphi_l, \quad l = 1, 2, \dots, n-1 \\ Y_{Bl} &= R \cdot \sin \varphi_l, \\ X_{Bm} &= L + R \cdot \cos \varphi_m, \quad m = 2, 3, \dots, n \\ Y_{Bm} &= R \cdot \sin \varphi_m, \end{aligned} \quad (4)$$

8. Using the condition for geometric constraint of the crank and the arrays (3) and (4), six arrays of the sinus and cosines of the angular orientations of the rocker ψ_k , ψ_l , ψ_m are determined.

$$\begin{aligned} \sin \psi_k &= \frac{b_k \cdot c_k + a_k \cdot \sqrt{a_k^2 + b_k^2 - c_k^2}}{a_k^2 + b_k^2}, \\ \cos \psi_k &= \frac{-a_k \cdot c_k + b_k \cdot \sqrt{a_k^2 + b_k^2 - c_k^2}}{a_k^2 + b_k^2}, \\ \sin \psi_l &= \frac{b_l \cdot c_l + a_l \cdot \sqrt{a_l^2 + b_l^2 - c_l^2}}{a_l^2 + b_l^2}, \\ \cos \psi_l &= \frac{-a_l \cdot c_l + b_l \cdot \sqrt{a_l^2 + b_l^2 - c_l^2}}{a_l^2 + b_l^2}, \\ \sin \psi_m &= \frac{b_m \cdot c_m + a_m \cdot \sqrt{a_m^2 + b_m^2 - c_m^2}}{a_m^2 + b_m^2}, \\ \cos \psi_m &= \frac{-a_m \cdot c_m + b_m \cdot \sqrt{a_m^2 + b_m^2 - c_m^2}}{a_m^2 + b_m^2}, \end{aligned} \quad (5)$$

where:

$$\begin{aligned} a_k &= 2R \cdot (L - X_{Ak}); \quad b_k = 2R \cdot Y_{Ak}; \quad c_k = L^2 + r^2 + R^2 + l^2 - 2L \cdot X_{Ak} \\ a_l &= 2R \cdot (L - X_{Al}); \quad b_l = 2R \cdot Y_{Al}; \quad c_l = L^2 + r^2 + R^2 + l^2 - 2L \cdot X_{Al} \\ a_m &= 2R \cdot (L - X_{Am}); \quad b_m = 2R \cdot Y_{Am}; \quad c_m = L^2 + r^2 + R^2 + l^2 - 2L \cdot X_{Am} \end{aligned}$$

9. Three arrays of the angular orientations of the rocker are calculated:

$$\begin{aligned} \psi_k &= 2 \cdot \arccos(\text{sign}(\sin \psi_k) \cdot \sqrt{(0,5 + 0,5 \cdot \cos \psi_k)}) \\ \psi_l &= 2 \cdot \arccos(\text{sign}(\sin \psi_l) \cdot \sqrt{(0,5 + 0,5 \cdot \cos \psi_l)}) \\ \psi_m &= 2 \cdot \arccos(\text{sign}(\sin \psi_m) \cdot \sqrt{(0,5 + 0,5 \cdot \cos \psi_m)}) \end{aligned} \quad (6)$$

10. By performing two consecutive numerical differentiations, two arrays of numerical values are determined, respectively for the first and second translation functions of the mechanism.

$$\psi'_R = \frac{\psi_m - \psi_k}{2h} \quad (7)$$

$$\psi''_R = \frac{\psi_m - 2\psi_l + \psi_k}{h^2} \quad (8)$$

11. Two arrays of the numerical values of the angular velocity ω_R and the angular acceleration ε_R of the rocker are determined.

$$\omega_R = \psi'_R \cdot \omega \quad (9)$$

$$\varepsilon_R = \psi''_R \cdot \omega^2 \quad (10)$$

12. Six arrays of the numerical values of the coordinates of the hinge B are calculated.

$$\begin{aligned}
 X_{Bk} &= L + R \cdot \cos \psi_k, \\
 Y_{Bk} &= R \cdot \sin \psi_k, \\
 X_{Bl} &= L + R \cdot \cos \psi_l, \\
 Y_{Bl} &= R \cdot \sin \psi_l, \\
 X_{Bm} &= L + R \cdot \cos \psi_m, \\
 Y_{Bm} &= R \cdot \sin \psi_m.
 \end{aligned} \quad (11)$$

13. Six arrays of the numerical values of the sinus and cosines of the angular orientations of the winch are determined

$$\begin{aligned}
 \sin \theta_k &= \frac{Y_{Bk} - Y_{Ak}}{l}, \\
 \cos \theta_k &= \frac{X_{Bk} - X_{Ak}}{l}, \\
 \sin \theta_l &= \frac{Y_{Bl} - Y_{Al}}{l}, \\
 \cos \theta_l &= \frac{X_{Bl} - X_{Al}}{l}, \\
 \sin \theta_m &= \frac{Y_{Bm} - Y_{Am}}{l}, \\
 \cos \theta_m &= \frac{X_{Bm} - X_{Am}}{l}.
 \end{aligned} \quad (12)$$

14. Three arrays of the numerical values of the angular orientations of the winch are calculated:

$$\begin{aligned}
 \theta_k &= 2 \cdot \arccos(\text{sign}(\sin \theta_k) \sqrt{(0,5 + 0,5 \cdot \cos \theta_k)}) , \\
 \theta_l &= 2 \cdot \arccos(\text{sign}(\sin \theta_l) \sqrt{(0,5 + 0,5 \cdot \cos \theta_l)}) , \\
 \theta_m &= 2 \cdot \arccos(\text{sign}(\sin \theta_m) \sqrt{(0,5 + 0,5 \cdot \cos \theta_m)}) .
 \end{aligned} \quad (13)$$

15. Two arrays of the numerical values of the first and second derivatives of the angular orientations of the winch are calculated, respectively.

$$\begin{aligned}
 \theta_l' &= \frac{\theta_m - \theta_k}{2h}, \\
 \theta_l'' &= \frac{\theta_m - 2\theta_l + \theta_k}{h^2}.
 \end{aligned} \quad (14)$$

16. Two arrays of numerical values of the angular velocity and angular acceleration of the object are defined.

$$\begin{aligned}
 \omega_R &= \theta_l' \cdot \omega, \\
 \varepsilon_R &= \theta_l'' \cdot \omega^2.
 \end{aligned} \quad (15)$$

17. Six arrays of numerical values of the coordinates of the center of gravity of the object are calculated. Here this will be done assuming that the object has a constant cross-section.. [3]

$$\begin{aligned}
 X_{Sk} &= \frac{Y_{Ak} + Y_{Bk}}{2}, \\
 Y_{Sk} &= \frac{X_{Bk} - X_{Ak}}{2}, \\
 X_{Sl} &= \frac{Y_{Al} + Y_{Bl}}{2}, \\
 Y_{Sl} &= \frac{X_{Bl} - X_{Al}}{2}, \\
 X_{Sm} &= \frac{Y_{Am} + Y_{Bm}}{2}, \\
 Y_{Sm} &= \frac{X_{Bm} - X_{Am}}{2}.
 \end{aligned} \quad (16)$$

18. Four arrays of numerical values are defined, two of which are for the velocity projections and two for the acceleration projections of the center of gravity S of the object. [4]

$$\begin{aligned}
 V_{Sx} &= \frac{X_{Sm} - X_{Sk}}{2t}, \\
 V_{Sy} &= \frac{Y_{Sm} - Y_{Sk}}{2t}, \\
 a_{Sx} &= \frac{X_{Sm} - X_{Sl} + X_{Sk}}{t^2}, \\
 a_{Sy} &= \frac{Y_{Sm} - Y_{Sl} + Y_{Sk}}{t^2}.
 \end{aligned} \quad (17)$$

19. Plot the graphs of the kinematic and geometric parameters of the mechanism as a function of the angular orientation of the leading knee 1.

III. RESULTS AND DISCUSSION

Example: To perform a kinematic analysis of a hinged four-link mechanism given: $\omega=80\text{s}^{-1}$, $r=60\text{mm}$, $L=120\text{mm}$, $\varphi_1=0$, $\varphi_0=\pi/6$.

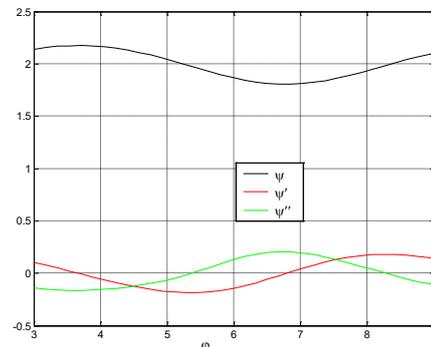


Fig.2 The change in the angle of the third link, as the first and second position functions.

The analysis was performed according to the compiled algorithm.

In Fig.2, the graphs of the position function $\psi_R = \psi_R(\varphi)$, of the first translation function $\psi'_R = \psi'_R(\varphi)$ and of the second translation function $\psi''_R = \psi''_R(\varphi)$ are plotted.

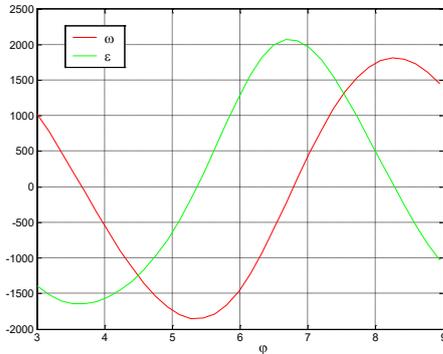


Fig.3 Angular velocity and angular acceleration.

In Fig.3, the graphs of the angular velocity and angular acceleration of link 2 are plotted.

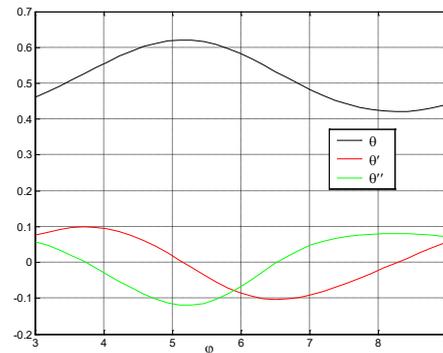


Fig.4 The relative rotation of the second unit

In Fig.4, the graphs of the angular orientations $\theta = \theta(\varphi)$, $\theta' = \theta'(\varphi)$ and $\theta'' = \theta''(\varphi)$ are plotted.

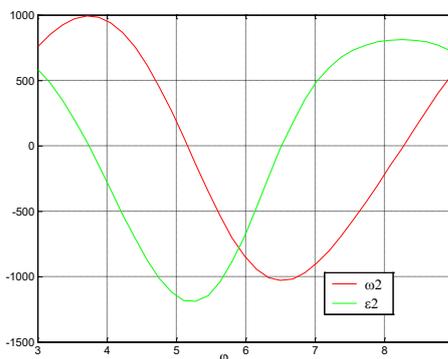


Fig.5 The angular velocity and angular acceleration of the second unit.

Fig.5 shows the graphs of the angular velocity and angular acceleration of the forklift.

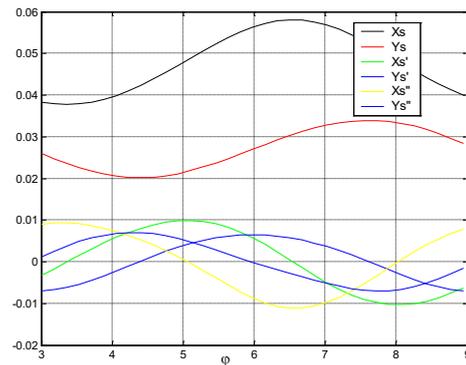


Fig.6 The coordinates of the centers of gravity of units 1, 2 and 3.

Fig.6 shows the graphs $X_s = X_s(\varphi)$ and $Y_s = Y_s(\varphi)$, as well as the first and second translation functions of the two displacements.

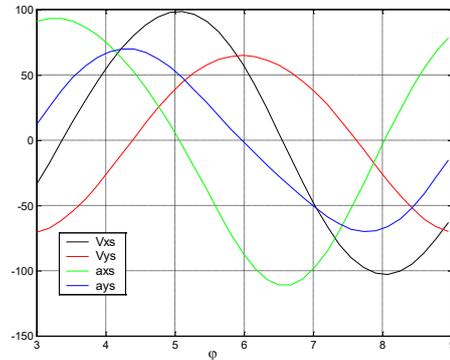


Fig.7. The linear velocities and linear accelerations of the centers of gravity of the initial unit.

Fig.7 shows the graphs of the velocities and accelerations of point S as a function of the rotation of the initial unit 1. [9]

IV. CONCLUSIONS

This article is intended for the education of bachelor's and master's students in the field of mechanical engineering and the field of applied analysis of mechanisms and machines for industry.

The purpose of this publication is to raise the level of teaching in the field of the theory of machine mechanisms and automatic lines in industrial production.

The program prepared in the MathLab 2020 environment is very voluminous and therefore only the graphs of the derivative functions (velocities and accelerations) are given.

The development of program modules for synthesis, in an environment of modern software products, allows

for the acceleration of the synthesis process itself, as well as for the analysis of the obtained results.

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VI. REFERENCES

- [1] A., Lengerov, D., Petrova and M., Bozhakov, Technological Assurance Of The Milling Process Of Prismatic Channels In Heat Exchange Aluminum Alloys, Vide. Tehnologija. Resursi - Environment, Technology, Resources, Volume 3, Pages 154 – 157, 2024, 15th International Scientific and Practical Conference on Environment. Technology. Resources, ETR 2024, Rezekne 27 June 2024 through 28 June 2024, ISSN 16915402, ISBN 978-171389957-0, DOI 10.17770/etr2024vol3.8163, ISSN 0094243X, 15517616.
- [2] D., Petrova, Vlahova, B., Lengerov, A. and Zlateva-Petkova, Improvement of Constructive-Technological Approaches Reducing Innovative Obsolescence of Industrial Technologies in the Context of Industry 4.0, Vide. Tehnologija. Resursi - Environment, Technology, Resources, Proceeding of the 14th International Scientific and Practical Conference, 2023, Volume I, pp. 180–186, Print ISSN 1691-5402, Online ISSN 2256-070X, DOI: <https://doi.org/10.17770/etr2023vol1.7238> <https://www.scopus.com/authid/detail.uri?authorId=56323816400>
- [3] D., Petrova, Vlahova, B., Lengerov, A. and Zlateva-Petkova, T., Study of the State of Innovation Development and Obsolescence in the Republic of Bulgaria of Companies from the Mechanical Engineering Sector, Vide. Tehnologija. Resursi - Environment, Technology, Resources, Proceeding of the 14th International Scientific and Practical Conference 2023, Volume I, pp. 187–194, Print ISSN 1691-5402, Online ISSN 2256-070X, DOI: <https://doi.org/10.17770/etr2023vol1.7239> <https://www.scopus.com/authid/detail.uri?authorId=56323816400>
- [4] D., Petrova, N., Nikolova and A., Lengerov, Modeling business process reengineering in the digital era of industry 4.0, 11th International Scientific Conference “TechSys 2022” – Engineering, Technologies and Systems, AIP Conf. Proc. 2980,070013-1–070013-7; <https://doi.org/10.1063/5.0185275>, Published by AIP Publishing. 978-0-7354-4814-8/\$30.00, pp. 070013-1 to 070013-7, ISSN 0094243X, 15517616, SJR=0,15.
- [5] D. Petrova, N., Nikolova and A., Lengerov, Organizational design of the management system of the reengineered enterprise - Design and implementation, 11th International Scientific Conference “TechSys 2022” – Engineering, Technologies and Systems, AIP Conf. Proc. 2980, 070012-1–070012-8; <https://doi.org/10.1063/5.0185274>, Published by AIP Publishing. 978-0-7354-4814-8/\$30.00, pp. 070012-1 to 070012-8, ISSN 0094243X, 15517616, SJR=0,15.
- [6] D. Petrova, Innovative Obsolescence in Mechanical Engineering and Singleness of Indicators, Vide. Tehnologija. Resursi - Environment, Technology, Resources, Volume 3, Pages 221 – 224, 2024, 15th International Scientific and Practical Conference on Environment. Technology. Resources, ETR 2024, Rezekne 27 June 2024 through 28 June 2024, ISSN 16915402, ISBN 978-171389957-0, DOI 10.17770/etr2024vol3.8123.
- [7] G.A. Timofeev, Theory of Mechanisms and Machines, Moscow Urude 2010.
- [8] G. Iliev and H. Hristov, “Modelling and simulation of electropneumatic positioning system including the length of pneumatic lines”, ETR, vol. 3, pp. 106–111, Jan. 2024, doi: 10.17770/etr2023vol3.7186.
- [9] G. Iliev and H. Hristov, Speed control pneumatic cylinders using high speed 2 port on/off valves with pulse width modulation. environment, technologies, resources. Proceedings of the International Scientific and Practical Conference. 3, (Jun. 2024), 95–100. DOI: doi.org/10.17770/etr2024vol3.8117.
- [10] I. I., Artobolevskiy, Theory of Mechanisms and Machines. Science, Moscow, 1967.
- [11] J. Uicker, G. Pennok and J. Shigley, Theory of Machines and Mechanisms, Fifth Edition, Oxford University Press 2017.
- [12] L.L. Harrisberger, An overview of structural synthesis of three-dimensional mechanisms, constructing science and technology of mechanical engineering, World, Moscow, 1965.
- [13] M., Marinov, Synthesis of planar mechanisms by extremely distant discrete positions. Dissertation abstract to get the scientific and educational degree "PhD" - TU Sofia 2002, pp.36.
- [14] M.J. Marinov, B.I. Stoychev, D.M. Guteva and M.V. Georgiev, Computer modeling and geometric analysis of Click-Clack mechanisms. "Mechanics of the machines" 2016 booklet 118 pp.35-39.
- [15] Michael Stanisic, Mechanisms and machines; Kinematics, Dynamics and synthesis, Stamford 2015.
- [16] Myszka David, Machines & Mechanisms: Applied Kinematic Analysis, 2011.
- [17] P.T. Stoyanov, Geometric precision synthesis of planar lever mechanisms. "Mechanics of the machines" 2002 p.56 vol. 33.
- [18] R.A. Rusev and V.B. Galabov, Structuring of rational planet-lever mechanism for passing the weft through the weaving machines. "Mechanics of machines" 2002 pp.26 booklet 33.
- [19] S. Singh, Theory of Machines: Kinematics and Dynamics, Pearson Education, India 2011
- [20] S. Stefanov, Theoretical mechanics, statics, kinematics and dynamics, Sofia 2014.
- [21] V. B. Bhandari, Design of Machine Elements Fifth Edition, McGraw Hill 2020.
- [22] V.B. Galabov, M. J. Marinov and N.B. Nikolov, Synthesis on a guiding MF at two points, respectively phase relation and double midpoint, "Mechanics of machines" 1999, booklet 26.
- [23] V.B. Galabov and M. J. Marinov, Synthesis of articulated four-link mechanism by discrete relative positions. "Mechanics of the machines", Varna, 2002.