Environment. Technology. Resources. Rezekne, Latvia
Proceedings of the 16th International Scientific and Practical Conference. 2025. Volume IV, 239-243

On the Influence of the Elastic Anisotropy of the Propeller on the Vibrations of Azimuth Thrusters

Yuliyan Minchev
Department of Mechatronics
Naval Academy N.Vaptsarov, Varna, Bulgaria
Varna, Bulgaria
julminchev@nvna.eu

Ivaylo Minchev
Department of Mechatronics
Naval Academy N.Vaptsarov, Varna, Bulgaria
Varna, Bulgaria
iminchev@nvna.eu

***Abstract*— A possibility for excitation of oscillations from the propeller of azimuth thrusters is analyzed.. The shaft is short, cantilever type. The propeller blades create elastic anisotropy of the propeller shaft with respect to the transverse stiffness. The influence of the elastic anisotropy of the propeller shaft on the oscillations of azimuth thrusters is studied . A mathematical model describing the oscillations is proposed, which is reduced to the Mathieu equation. It is proven that the elastic anisotropy of the propeller shaft excites two types of oscillations: forced with a blade frequency and multiples of it, and parametric with a frequency equal to half the blade frequency and multiples of it. Experimental results are presented.**

***Keywords*— propeller, oscillations, elastic anisotropy, azimuth thrusters.**

I. Introduction

The vibrations excited by the propeller are the subject of attention of many researchers. Attention is focused on the hydrodynamic forces. As is known, they depend on the operating conditions of the propeller. (propeller behind the stern or propeller in the nozzle). The propeller is subject to static balancing of the inertial forces, which are a significant exciter of vibrations. A major exciter of vibrations is the hydrodynamic unbalance of the propeller. It is the result of a disturbed geometry of the propeller, which leads to a mismatch of the center of mass of the attached mass of water with the axis of rotation.

The oscillations of the ship's hull excited by the propeller are discussed in [1][2][3][6][7] , with experimental results presented in [3][6] and [7].

II. Materials and methods

In this work, a possibility for excitation of oscillations by the propeller of azimuth thrusters is analyzed . In this case, the propeller shaft is of the cantilever type (Fig. 1). The shaft is short. The propeller blades create elastic anisotropy of the shaft with respect to the transverse stiffness. As is known [4][5], the elastic anisotropy of the crankshaft of the main engines is an exciter of parametric oscillations of the shaft line. Therefore, in this work, the influence of the elastic anisotropy of the propeller shaft on the oscillations of azimuth thrusters is studied . In this case, the elastic anisotropy is due to the shape of the propeller. In the section where there is a blade, the transverse stiffness is greater. Fig. 2 shows a schematic of a shaft with a four-bladed propeller. It is not difficult to see that the transverse stiffness is a function of the angle of rotation φ . Along the x and axes, y the stiffness is maximal, while along the axis z , which corresponds to $\varphi = 45^o$, it is minimal. The transverse stiffness in this case can be approximated by a function of the form

$$C = C_O + C_1 \cos 4\varphi \quad (1)$$

The minimum stiffness is C_O , and the maximum is $C_O + C_1$.

We will compile a mathematical model describing the oscillations of a cantilevered comb propeller. The propeller is represented as a disk (Fig. 3), which rotates with a constant angular velocity ω . The propeller oscillates in the direction of the axes x and y . In addition, the angular oscillations α and β about the axes x and are also taken into account y . When compiling the mathematical model, we will apply the theorems for changing the amount of motion and the kinetic moment. The kinetic moment of rotation (Fig. 3) is:

$$K = I\omega \quad (2\text{-a})$$

Parameter I is the mass moment of inertia about the axis of rotation, which is orthogonal to the plane of the disk. The angular momentums α and β (Fig. 3) are:

Online ISSN 2256-070X
https://doi.org/10.17770/etr2025vol4.8435

$$K_x = I_x\dot{\alpha}\,, K_y = I_y\dot{\beta} \tag{2-b}$$

Here I_x and I_y are the equatorial mass moments of inertia of the disk with which the propeller is modeled.

As a result of the angular oscillations of the disk about the axes x and y the kinetic moment K of rotation, which is orthogonal to the plane of the disk, there is a projection (Fig. 4) along the fixed axis x:

$$I\omega\alpha \tag{3}$$

And projection(Fig. 4) along the fixed axis y:

$$I\omega\beta \tag{4}$$

Based on (1), (2), (3) and (4) and the graphical interpretation in Fig. 4, the kinetic moments along the axes x and are obtained y:

$$\begin{aligned} K_{OX} &= I_x\dot{\alpha} + I\omega\beta \\ K_{OX} &= I_y\dot{\beta} + I\omega\alpha \end{aligned} \tag{5}$$

We will define the elastic constants of the cantilever shaft [8] . When a displacement is given y (Fig. 5-a) of the free end, an elastic force arises along the axis y:

$$F_{yy} = C_{11}y \tag{6}$$

and moment on x:

$$M_{xy} = C_{21}y \tag{7}$$

When a displacement is set x (Fig. 5) at the free end, an elastic force arises along the axis x:

$$F_{xx} = C_{11}x \tag{8}$$

and moment about axis y:

$$M_{yx} = C_{21}x \tag{9}$$

When defining an angular deformation α (Fig. 5-c) along the axis x, an elastic force arises along the axis y

$$F_{yx} = C_{12}\alpha \tag{10}$$

and axial moment x

$$M_{xx} = C_{22}\alpha \tag{11}$$

When an angular deformation β (Fig. 5-d) is imposed along the axis y, an elastic force arises along the axis x

$$F_{xy} = C_{12}\beta \tag{12}$$

and axial moment y

$$M_{yy} = C_{22}\beta \tag{13}$$

Based on (6), (8), (10) and (12), after applying the momentum change theorem, the differential equations are obtained:

$$\left| \begin{aligned} m\ddot{x} + C_{11}x - C_{12}\beta = 0 \\ m\ddot{y} + C_{11}y - C_{12}\alpha = 0 \end{aligned} \right. \tag{14}$$

Based on (5), (7), (9), (11) and (13) after applying the theorem for changing the angular momentum, the differential equations are obtained:

$$\left| \begin{aligned} I_x\ddot{\alpha} + I\omega\dot{\beta} + C_{21}y - C_{22}\alpha = 0 \\ I_y\ddot{\beta} - I\omega\dot{\alpha} - C_{21}x + C_{22}\beta = 0 \end{aligned} \right. \tag{15}$$

Equations (14) and (15) form a system of equations that describe the oscillations of the propeller.

The elastic constants [8] have the form:

$$C_{11} = \frac{12EJ}{l^3}\,,\ C_{12} = \frac{6EJ}{l^2}\,, C_{22} = \frac{4EIJ}{l} \tag{16}$$

The elastic constants represent the transverse stiffnesses of the cantilever shaft with a comb screw. Due to the elastic anisotropy resulting from the presence of the blades, they are approximated by functions of the form (1)

$$C_{ij} = C_{ij}^o + C_{ij}^1 \cos k\varphi \tag{17}$$

We will limit ourselves to considering the oscillations only along the x and axes y, neglecting the angular oscillations. Then from (14) we obtain:

$$\begin{aligned} m\ddot{x} + C_{11}x = 0 \\ m\ddot{y} + C_{11}y = 0 \end{aligned} \tag{18}$$

The equations (18) are the same, so we will analyze the oscillations in x. Substituting (17) into (18) we obtain:

$$m\ddot{x} + \left(C_{11}^o + C_{11}{}' \cos k\varphi\right)x = 0 \tag{19}$$

Assuming $\sin k\varphi \approx \sin k\omega t$ we write equation (19) in the form

$$\ddot{x} + \omega_C^2\left(1 + \frac{C_{11}{}'}{C_{11}^o}\cos k\omega t\right)x = 0 \tag{20}$$

Here $\omega_C^2 = \dfrac{C_{11}^o}{m}$

After fitting, $k\omega t = \tau$ equation (20) takes the form

$$\frac{d^2x}{d\tau^2} + \left(\lambda + \mu\cos\tau\right)x = 0\,, \tag{21}$$

Where

$$\lambda = \left(\frac{\omega_C}{k\omega}\right)^2, \mu = \frac{\omega_C^2}{\left(k\omega\right)^2}.\frac{C_{11}^1}{C_{11}^o} = \frac{\lambda C_{11}^1}{C_{11}^o}$$

The resulting differential equation is the well-studied Mathieu equation. The type of the general solution of the equation depends on the values of the parametric λ and μ [9] . For small values of μ at

$$\lambda_1 \approx 1, \lambda_2 \approx 4$$

oscillations are excited with a frequency equal to the frequency of change of the stiffness and multiples of it:

$$\begin{aligned} x = A_1\sin k\omega t + B_1\cos k\omega t + \\ + A_2\sin 2k\omega t + B_2\cos 2k\omega t + ... \end{aligned}$$

For small values of μ, when

$$\lambda_1 \approx \frac{1}{4}, \lambda_2 \approx \frac{9}{4}$$

Parametric oscillations are excited with a frequency equal to half the frequency of stiffness change and multiples of it:

$$x = A_{\frac{1}{2}} \sin\frac{k\omega t}{2} + B_{\frac{1}{2}} \cos\frac{k\omega t}{2} +$$
$$+ A_{\frac{3}{2}} \sin\frac{3}{2}k\omega t + B_{\frac{3}{2}} \cos\frac{3}{2}k\omega t + ...$$

III. Results and discussion

The experimental results are from the two azimuth thrusters on a ship with a propeller rotation frequency of 244 *rpm* . The propellers are four-bladed. Forced oscillations with frequencies, $2.4\omega = 1952rpm$ are observed $4\omega = 976rpm$.

Fig. 6 presents results for the azimuth thruster - port side, and Fig. 7 for starboard side as shown in Table 1.

Table 1

direction	AT. port side	AT. starboard
vertically	Fig. 6-a	Fig. 7-a
horizontally	Fig. 6-b	Fig. 7-b
axial	Fig. 6-c	Fig. 7-c

It should be noted that the oscillations are of low intensity (up to $1{,}55 mm/s$). Here the qualitative result is essential, confirming the hypothesis of excitation of oscillations as a result of the elastic anisotropy of the propeller shaft with a propeller.

In the case under consideration, the screws are in a nozzle, therefore there is no hydrodynamic exciter of blade frequency oscillations.

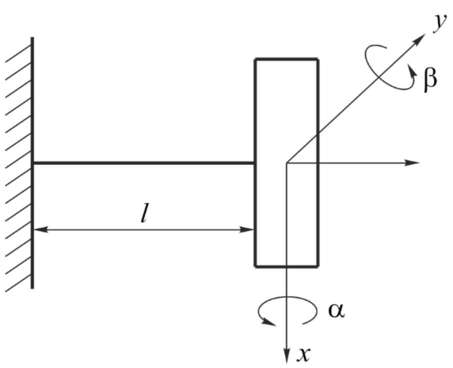


Fig. 1. The propeller shaft is of the cantilever type

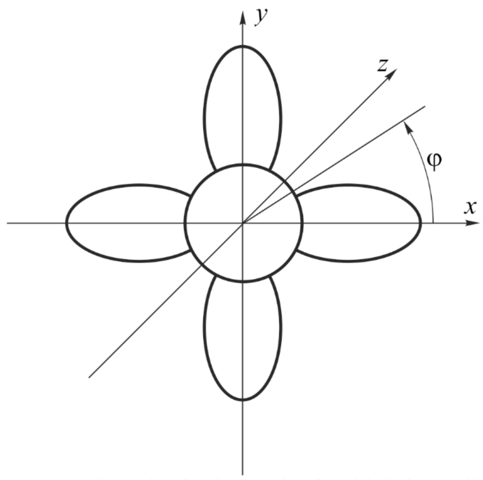


Fig. 2. A schematic of a shaft with a four-bladed propeller

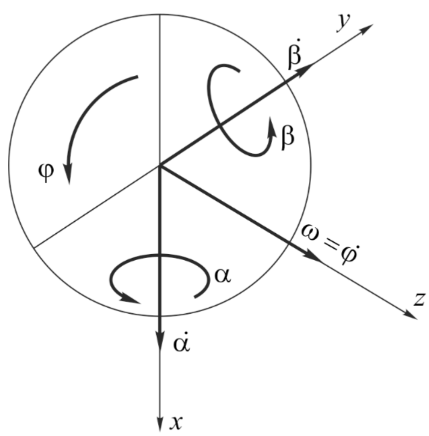


Fig. 3. The propeller is represented as a disk

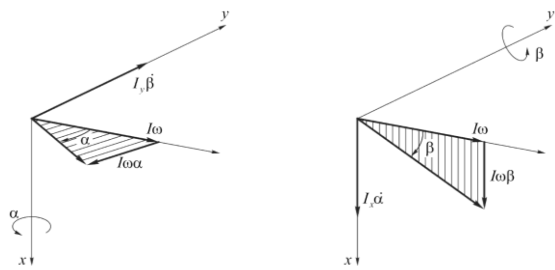
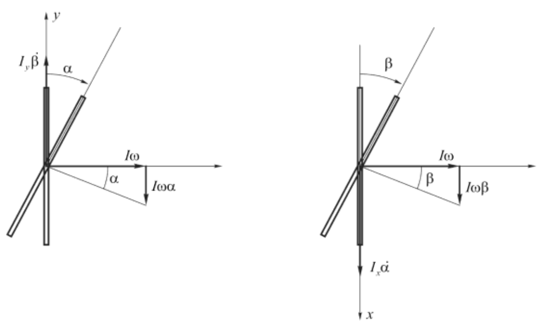


Fig. 4-a. Projection of the kinetic moment along the x and y axes



Fig, 4-b. Projection of the kinetic moment along the x and y axes

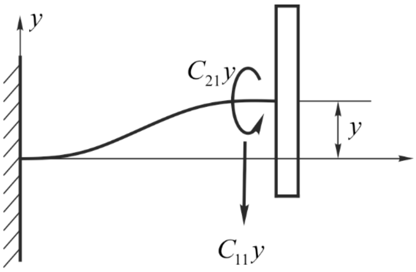


Fig. 5-a. the elastic constants of the cantilever shaft. When specifying a displacement y

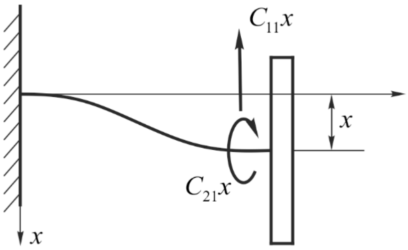


Fig. 5-b the elastic constants of the cantilever shaft. When specifying a displacement x

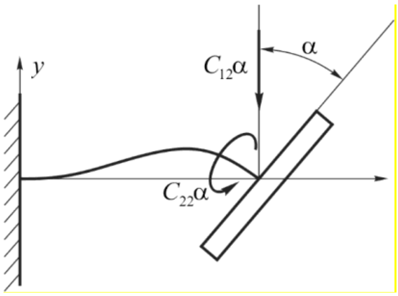


Fig. 5-c the elastic constants of the cantilever shaft. When specifying a displacement α

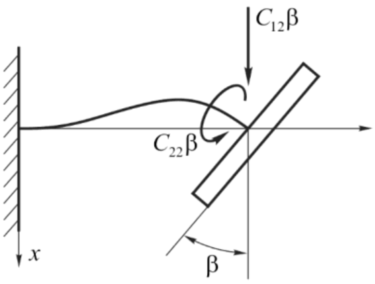


Fig. 5-d. the elastic constants of the cantilever shaft. When specifying a displacement β

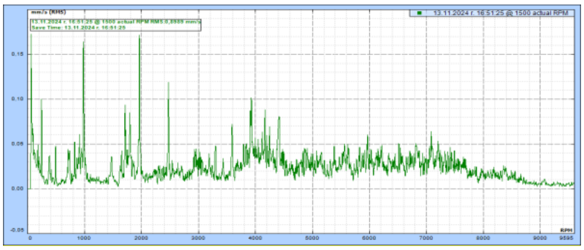


Fig. 6-a. Azimuth thruster oscillation spectrum port side vertical

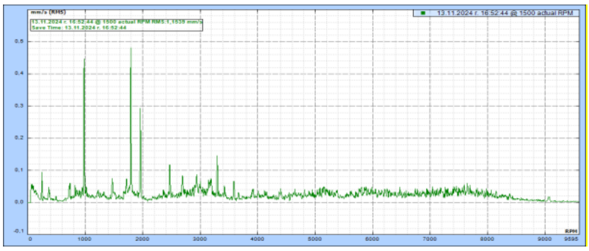


Fig. 6-b. Azimuth thruster oscillation spectrum port side horizontal

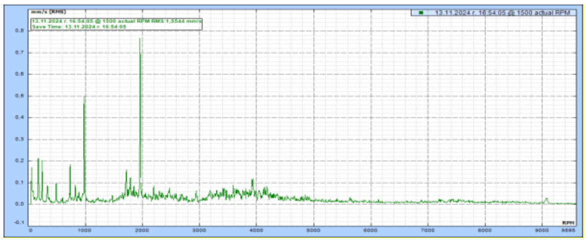


Fig. 6-c. Azimuth thruster oscillation spectrum port side axial

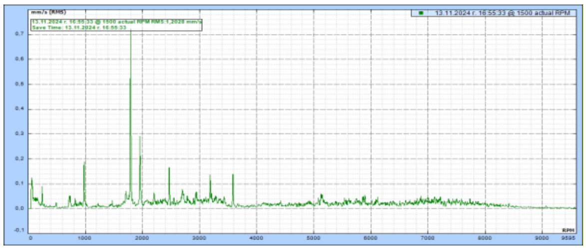


Fig. 7-a Azimuth thruster oscillation spectrum starboard side vertical

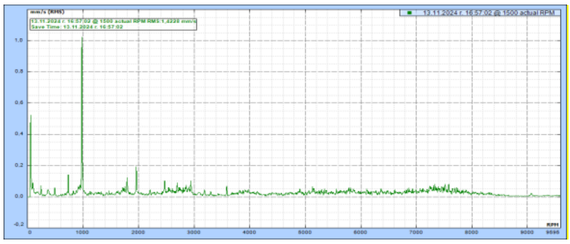


Fig. 7-b Azimuth thruster oscillation spectrum starboard side horizontal

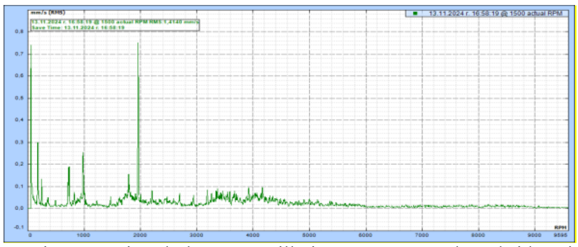


Fig. 7-c Azimuth thruster oscillation spectrum starboard side axial

REFERENCES

[1] N.D.Minchev, Dynamics of ship engines. DVI. Sofia 1983. p. 448.

[2] N.D.Minchev, Hr.Draganchev, Strength and vibrations of ship engines. IK. Galaktika 1992. p. 389.

[3] N.D. Minchev, Y.U. Minchev, I.N. Minchev, On the excitation of ship hull vibrations by the propeller. Journal Mechanics of Machines No. 107, pp. 81-91.

[4] N.N. Minchev, Y.U. Minchev, I.N. Minchev, Transverse vibrations of a ship's shaft line and a ship's engine with

consideration of the elastic anisotropy of the crankshaft. Journal of Machine Mechanics No. 42 pp. 88-100.

[5] N.N. Minchev, Y.U. Minchev, I.N. Minchev, Mechanomathematical modeling of transverse vibrations of a ship shaft line with consideration of the elastic anisotropy of the crankshaft. Journal Mechanics of Machines No. 49, pp. 165-171.

[6] Y.M. Minchev, I.N. Minchev, Mechanomathematical model and experimental results on the excitation of ship hull vibrations by a paddle wheel. Journal Mechanics of Machines No. 126, pp. 158-168.

[7] N.D. Minchev, Y.U. Minchev, I.N. Minchev, On the excitation of ship hull vibrations by the propeller. Journal of Machine Mechanics No. 107, 2014, pp. 81-90.

[8] F.M. Dimentberg, K.T. Shatalov, A.A. Gusarov, The wobbly machine . Ed. Mechanical Engineering , Moscow 1964.

[9] N.D. Minchev V. Grigorov, Vibrodiagnostics of rotary and piston machines. DI "Technika" Sofia 1988. p. 280