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Influence of the Geometric Eccentricity of a Gear on the Vibrations of a Bevel Gear

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***Abstract*— The influence of geometric eccentricity on the oscillations of bevel gears is studied. A gear train consisting of two "exact" gears with axes offset from the geometric center is considered. A mathematical model is compiled through which the oscillations in the tooth engagement are studied. It is proved that the oscillations in the contact zone are described by the Mathieu equation. It turns out that the geometric eccentricity of bevel gears excites two types of oscillations - forced and parametric. Experimental results from a ship's power system are presented.**

***Keywords*— oscillations, bevel gears, geometric eccentricity, azimuth thrusters.**

I. INTRODUCTION

Gear vibration has been the subject of much attention by many authors. The variable stiffness of the gear mesh is a major natural vibration exciter [1][2][3]. It excites two types of vibration [1] :

1. Forced oscillations with a frequency equal to the frequency of stiffness change and multiples of it

$$z\omega \quad , \quad 2z\omega \quad , \quad 3z\omega \quad ,\dots$$

here z is the number of teeth, and ω is the rotation frequency.

2. Parametric oscillations with a frequency equal to half the frequency of stiffness change and multiples of it:

$$\frac{1}{2}z\omega\,,\ \frac{3}{2}z\omega\,,\ \frac{6}{2}z\omega\,,\ \dots.$$

Vibrations are also caused by technological errors in the manufacture of gears:

- error in the main step [4][5] ;
- profile error [1];
- geometric eccentricity of the gears [1][7] .

II. MATERIALS AND METHODS

The influence of geometric eccentricity on the vibrations of cylindrical gears was studied in [1][7] . In the present work, the influence of geometric eccentricity on the vibrations of bevel gears (Fig. 1) is studied. We will consider a gear consisting of two “exact” gears (Fig. 2) with axes displaced from the geometric center. The centers of the base circles of the wheels are located at the points O' and O'' . The common normal $\tau-\tau$ to the base circles is the normal to the tooth profiles at the contact point. In the presence of eccentricity, the point O' and O'' rotate about the points O_1 and O_2 . This causes a displacement of the line $\tau-\tau$, a displacement of the engagement pole and a change in the gear ratio. For cylindrical gears with geometric eccentricity, this problem was studied in [1] . In Fig. 2 , the centers of rotation of the wheels are marked with O_1 and O_2 . They are displaced at a distance $l_1 = O_1O'$ and $l_2 = O_2O''$ from the geometric centers. From the centers of rotation O_1 and O_2 we draw the normals O_1A_1 and O_2B_1 to the common normal $\tau-\tau$ of the base circles. Therefore, the lines O_1A_1 and O_2B_1 are parallel to the radii of the base circles, respectively.

$$r_1 = O'A'\,, r_2 = O''B'$$

For the segments O_1A_1 and O_2B_1 is satisfied

$$\begin{aligned} O_1A_1 &= r_1 + e_1\cos(\varphi_1+\alpha) = r_{O1} \\ O_2B_1 &= r_2 + e_2\cos(\varphi_2-\alpha) = r_{O2} \end{aligned} \tag{1}$$

For the translation relation we get

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$$i = \frac{\omega_1}{\omega_2} = \frac{r_2 + e_2 \cos(\varphi_2 - \alpha)}{r_1 + e_1 \cos(\varphi_1 + \alpha)} = \frac{r_2}{r_1} \cdot \frac{1 + \frac{e_2}{r_2} \cos(\varphi_2 - \alpha)}{1 + \frac{e_1}{r_1} \cos(\varphi_1 + \alpha)} = i_O \frac{1 + \lambda_2 \cos(\varphi_2 - \alpha)}{1 + \lambda_1 \cos(\varphi_1 + \alpha)} \quad (2)$$

Here $i_O = \frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}$ - translational relationship of precisely mounted wheels.

$$\lambda_1 = \frac{e_1}{r_1}, \lambda_2 = \frac{e_2}{r_2} \quad (3)$$

The magnitudes of λ_1 and λ_2 are small. Therefore, the translation relation (2) is of the form(3)

$$i = i_O(1+\varepsilon),$$

Where ε is a small parameter.

After developing (2) in order, we obtain

$$\varepsilon = \lambda_2 \cos(\varphi_2 - \alpha) - \lambda_1 \cos(\varphi_1 + \alpha) \quad (4)$$

Here $\varphi_1 = \omega_1 t$, $\varphi_2 = \omega_2 t$

We will build a mathematical model of the single-stage bevel gear. The dynamic model is shown in Fig. 3. The gear consists of two gears 1 and 2, mounted on elastic supports with stiffnesses C_1 and C_2 in the direction of the meshing line. The stiffness of the gear mesh is represented by an elastic element with stiffness C also in the direction of the meshing line. The angular coordinates of the wheels 1 and 2 are φ_1 and φ_2, and their vibrational displacements in the direction of the meshing line are denoted by y_1 and y_2. The line $\tau - \tau$ performs small oscillations around an average position, which can be neglected when determining the direction of the axes y_1 and y_2.

The deformation of the tooth occlusion is

(5) $x = r_{O1}\varphi_1 + r_{O2}\varphi_2 + y_1 - y_2$

Based on (3) we present

$$r_{O1} = r_1\left(1 + \lambda_1 \cos(\omega_1 t + \alpha)\right)$$
(6)
$$r_{O2} = r_2\left(1 + \lambda_2 \cos(\omega_2 t - \alpha)\right)$$

After applying the Lagrange method, the equations are obtained

$$\begin{cases} I_1\ddot{\varphi}_1 + Cxr_{O1}(t) = 0 \\ I_2\ddot{\varphi}_2 + Cxr_{O2}(t) = 0 \\ m_1\ddot{y}_1 + C_1 y_1 + Cx = 0 \\ m_2\ddot{y}_2 + C_2 y_2 - Cx = 0 \end{cases} \quad (7)$$

In (7) I_1 and I_2 denote the mass moments of inertia of the gears, and m_1 and denote m_2 their masses. Equations (7) represent a system of differential equations with variable coefficients describing the oscillations of eccentrically mounted “precise” bevel gears.

We will study the oscillations in the gear meshing of eccentrically mounted “precision” gears. For this purpose, we assume $y_1 \equiv y_2 \equiv 0$. From (7) we obtain

$$\begin{cases} I_1\ddot{\varphi}_1 + Cxr_{O1}(t) = 0 \\ I_2\ddot{\varphi}_2 + Cxr_{O2}(t) = 0 \end{cases} \quad (8)$$

We substitute (6) into (8) and write (8) in the form:

$$\begin{cases} \ddot{\varphi}_1 + \frac{C}{I_1} r_1 x + \frac{1}{I_1} xCr_1\lambda_1 \cos\psi_1 = 0 \\ \ddot{\varphi}_2 + \frac{C}{I_2} r_2 x + \frac{1}{I_2} xCr_2\lambda_2 \cos\psi_2 = 0 \end{cases} \quad (9)$$

Here $\psi_1 = \omega_1 t + \alpha$, $\psi_2 = \omega_2 t - \alpha$.

With accuracy up to small first-order quantities, we assume

$$x = r_1\varphi_1 + r_2\varphi_2 \quad (10)$$

We will multiply the first equation of (9) by r_1, and the second by r_2 and sum them. Thus we get

$$\ddot{x} + \omega_C^2(1 + \varepsilon_1 \cos\psi_1 + \varepsilon_2 \cos\psi_2)x = 0 \quad (11),$$

where

$$\omega_C^2 = C\left(\frac{r_1^2}{I_1} + \frac{r_2^2}{I_2}\right), \ \varepsilon_1 = C\frac{r_1^2}{I_1}\lambda_1\frac{1}{\omega_C^2},$$

$$\varepsilon_2 = C\frac{r_2^2}{I_1}\lambda_2\frac{1}{\omega_C^2}$$

Equation (11) describes the vibrations in the contact zone in the general case when both wheels have geometric eccentricity.

When the second wheel is exactly $\lambda_2 = 0$. The vibrations in the contact area are described by the equation:

$$\ddot{x} + \omega_C^2(1 + \varepsilon_1 \cos\psi_1)x = 0 \quad (12)$$

When the first wheel is straight, $\lambda_1 = 0$ the vibrations in the contact patch are described by the equation

$$\ddot{x} + \omega_C^2(1 + \varepsilon_2 \cos\psi_2)x = 0 \quad (13)$$

We will present (12) and (13) in general form

$$\ddot{x} + \omega_C^2 (1 + \varepsilon_i \cos\psi_i) x = 0 \qquad (14)$$
$$i = 1, 2$$

From (14) we get

$$\frac{d^2x}{d\psi_i^2} + (\lambda_i + \mu_i \cos\psi_i) x = 0 \,, \qquad (15)$$

Where

$$\lambda_i = \left(\frac{\omega_C}{\omega_i} \right)^2, \mu_i = \left(\frac{\omega_C}{\omega_i} \right)^2 \varepsilon_i$$

This is Mathieu's equation. The equation has two types of solutions,

$$x = A_1 \sin\omega_i t + B_1 \cos\omega_i t + + A_2 \sin 2\omega_i t + B_2 \cos 2\omega_i t ... \qquad (16)$$

$$x = A_{\frac{1}{2}} \sin\frac{\omega_i}{2} t + B_{\frac{1}{2}} \cos\frac{\omega_i}{2} t + + A_{\frac{3}{2}} \sin\frac{3}{2}\omega_i t + B_{\frac{3}{2}} \cos\frac{3}{2}\omega_i t ... \qquad (17)$$

Solution (16) defines forced oscillations with a wheel rotation frequency of geometric eccentricity and multiples of it.

Solution (17) defines parametric oscillations with a frequency equal to half the wheel rotation frequency with geometric eccentricity and multiples thereof.

The values of μ_i are small. In such a case, at

$\lambda_i \approx 1$ and $\lambda_i \approx 4$

forced oscillations of the type (16) are observed.

At small values of μ_i in the vicinity of

$$\lambda_i \approx \frac{1}{4}, \lambda_i \approx \frac{9}{4}$$

parametric oscillations of the type (17) are observed.

It turns out that the geometric eccentricity of bevel gears excites two types of oscillations:

- forced by the species(16)
- parametric of the type (17)

An analogous result is defined in [1] for spur gears.

Vibrations in the contact area of bevel gears excite radial and axial vibrations of the bearings.

III. RESULTS AND DISCUSSION

We will present experimental results on the vibrations of the driving bevel gear of a main engine of a ship's propulsion system. The ship is a diesel- electric motor . The engine is asynchronous with a power of 1500 kW at 1791 rpm. Fig. 4 presents the spectra of the vibrations of the driving bevel gear in the three directions:

- vertical (Fig. 4-a) - 2,029 mm/s
- horizontal (fig. 4-b)-5.3712 mm/s
- axial (Fig. 4-c) - 3.0698 mm/s

In the vertical and horizontal vibrations, the vibrations with rotational frequency (1791rpm) are dominant. These are forced oscillations. In the axial oscillations, the oscillations with the frequency are dominant

$\frac{1}{2}\omega_1 \approx 900 rpm$. These are parametric oscillations. It

should be noted that the frequency range in which parametric oscillations are observed is narrow. In this case, the engine is asynchronous, therefore the rotation frequency is practically constant. This creates conditions for the existence of parametric oscillations continuously.

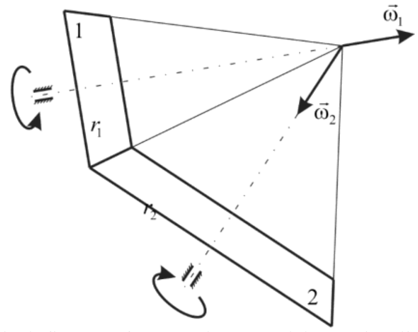
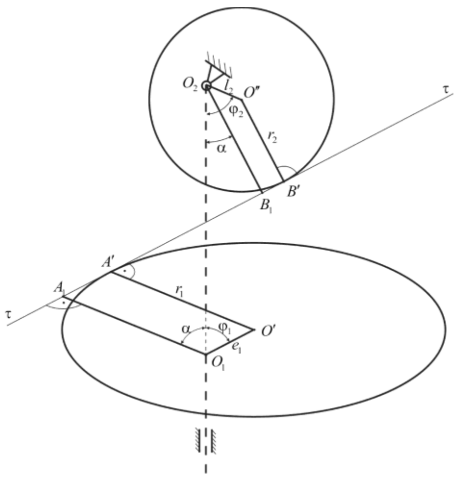


Fig. 1. The influence of geometric eccentricity on the vibrations of bevel gears



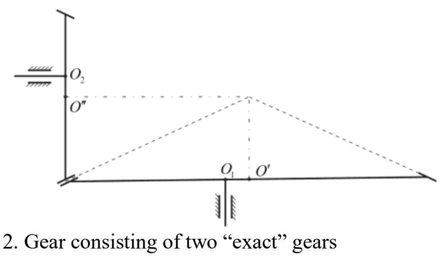


Fig. 2. Gear consisting of two "exact" gears

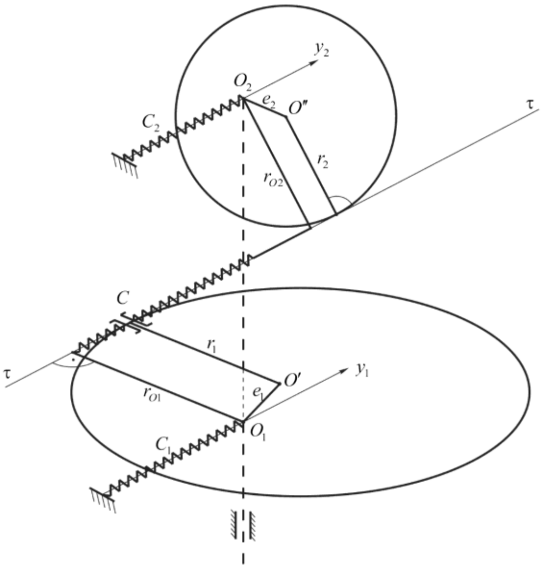


Fig. 3. The dynamic model of the single-stage bevel gear

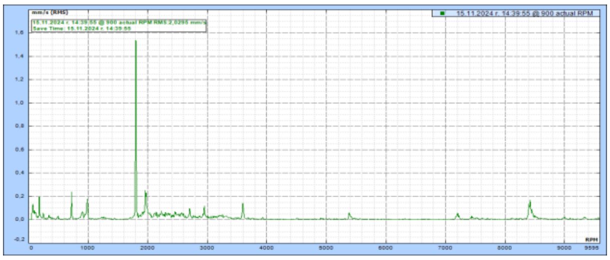


Fig. 4-a. The spectra of the vibrations of the driving bevel gear vertical

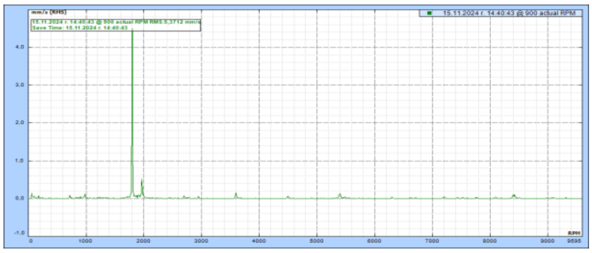


Fig. 4-b. The spectra of the vibrations of the driving bevel gear horizontal

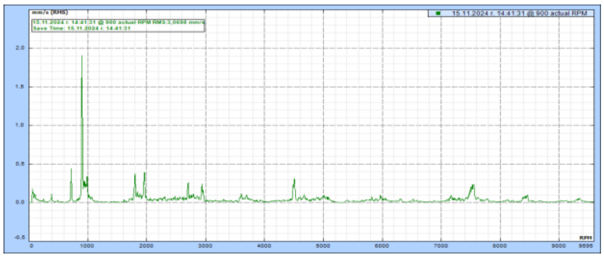


Fig. 4-c. The spectra of the vibrations of the driving bevel gear axial

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